# 3rd International Conference on Applied Mathematics and Computer Science (ICAMCS'23).


## Organized by
## Modeling and Combinatorics Laboratory, Polydisciplinary Faculty of Safi Conference Room

**From 20 to 22 June 2023**



icamcs.uca.ma

# Welcome Address

In behalf of organization committee, we would like to welcome you to the 3rd International Conference on Applied Mathematics and Computer Science ICAMCS'23. This conference will take place at the Poly-disciplinary Faculty in Safi, from June 20 to 22, 2023. Presentations will focus on Stochastic Calculus, Dynamic Systems, operation research and Computer Science. The aim of this meeting is to bring together and to foster exchanges and collaborations among scientists working in the field of applied mathematics and computer science, including those listed above. This event consists also of bridging industrials with mathematicians and computer scientist through highlighting topics of interest to socio-economic sector and those dealing with development Participants will also have the opportunity to visit the city of Safi, described by Ibn Khaldoun as the surrounding sea, which has a long and rich history.

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# Participant List

# Participant List

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| El abdelaoui | Said | Cadi Ayyad University | |
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| Sammad | Khalil | Cadi Ayyad University | |

# Conference Program

| Tuesday, June 20, 2023 | | |
|---|---|---|
| **08h30** | **Registration** | |
| **09h00** | **Opening ceremony** | |
| **From 09h15 to 09h55** | **Conference 1: S. Hamadène** | |
| **From 10h to 10h40** | **Conference 2 : Y. Belhamadia** | |
| **From 10h40 to 11h10** | **Coffee break** | |
| | **Parallel sessions** | |
| | **Stochastic Calculus, Dynamical system & control theory** | **Numerical Methods, discrete mathematics & Computer Science** |
| **From 11h10 to 12h25** | **11h10 - 11h35 : A. Essarhir**<br><br>**Title** : Semilinear diffusion with multiplicative noise | **11h10 - 11h30 : M. Mabdaoui**<br><br>**Title:** Finite element method for elliptic problems involving the operators satisfying non-polynomial growth. |
| | **11h40 - 12h : Badr ELMANSOURI**<br><br>**Title** : Pricing Americain game options in Azéma's markets: A doubly reflected BSDEs with RCLL martingales approach | **11h35 - 11h55 : Laila Loudiki**<br><br>**Title** : L(2,1)-labeling number and upper traceable number of circulant graphs |
| | **12h05 - 12h25 : BOUGGAR Driss**<br><br>**Title** : Sufficient and Necessary Conditions for Stochastic Near-Optimal Control in a Within-Host Infectious Disease Model | **12h - 12h20 : Maimouna Lapointe**<br><br>**Title** : Review of articles on Automatic Arabic diacritization |
| | **12h30 - 12h50 : Charaf-eddine Rhazlane**<br><br>**Title :** Backward Stochastic Evolution Inclusions in UMD Banach Spaces | **12h25 - 12h45 : Jaouad CHAOUI**<br><br>**Title :** Mathematical Modeling and Analysis of Micropolar Fluid Flow with Frictionless Contact Boundary Conditions |
| **From 12h50 to 15H00** | **Lunch break** | |
| **From 15h00 to 15h40** | **Conference 3: E. Eberlein** | |
| **From 15h40 to 16h10** | **Coffee break** | |
| | **Parallel sessions** | |
| | **Stochastic Calculus, Dynamical** | **Numerical Methods, discrete** |

| | system & control theory | mathematics & Computer Science |
|---|---|---|
| **From 16h10 to 18h20** | **16h10 – 16h35 : Mohamed El Omari**<br><br>**Title :** A class of Gaussian Volterra processes as extensions of the fractional Brownian motion | **16h10 – 16h35 : Kchikech Mustapha**<br><br>**Title :** Packing chromatic number of iterated Mycielskians |
| | **16h40 – 17h : Soukaina AIT YOUSEF**<br><br>**Title :** Geometric approach of a product form stationary distribution for an SRBM in three dimensions | **16h40 – 17h : OUAANABI Abdelhafid**<br><br>**Title :** Analysis results for dynamic contact problem thermopiezoelectric materials |
| | **17h05 – 17h30 : Lakbir Essafi**<br><br>**Title :** Optimal control of a contact problem in Orlicz spaces | **17h05 – 17h25 : Omar EL GHATI**<br><br>Title : A brief overview of the applications of AI-powered Visual IoT systems in agriculture |
| | **17h35 – 17h55 : Mohamed El Hathout**<br><br>**Title :** Structure of positive radial solutions of a nonlinear boundary value problem including the p-Laplacian operator | **17h30 – 17h55 : Issam MATAZI**<br><br>**Title :** The Application of Machine Learning in E-learning |
| | **18h - 18h20 : Abdelati Lagzini**<br><br>**Title :** Bayesian inference for SIS type epidemic model, by Skellam's distribution and application to COVID | **18h - 18h20 : Mourdi Youssef**<br><br>**Title :** MOOC's Learners classification : A behavioral generation framework based methodology |

## Wednesday, June 21, 2023

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|---|---|
| **From 09h00 to 09h40** | **Conference 1 : J. Koko** |
| **From 09h45 to 10h25** | **Conference 2 : K. Hamamache** |
| **From 10h30 to 11h00** | **Coffee break** |

**Parallel sessions**

| | Stochastic Calculus, Dynamical system & control theory | Numerical Methods, discrete mathematics & Computer Science |
|---|---|---|
| **From 11h00 to 12h15** | **11h00 - 11h25 : Hafida Atti**<br><br>**Title :** LU Decomposition Method to Solve Intuitionistic Fuzzy Linear Systems | **11h00 - 11h25 : Bouchra Ben Amma**<br><br>**Title :** A Comparative Study of Numerical Techniques for Solving Intuitionistic Fuzzy Differential Equations |
| | **11h30 - 11h 50: Tarik Aslaoui** | **11h30 - 11h 50 : Abdelaaziz BELLOUT** |

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|---|---|---|
| | **Title :** Solving higher order intuitionistic fuzzy differential equations | **Title :** Deep Learning approach for tomato leaf disease prediction |
| | **11h55 - 12h15 : Ilham Ouelddris**<br><br>**Title :** Null approximate impulse controllability for parabolic degenerate singular equations via logarithmic convexity | **11h55 - 12h15 : Aziza BARAKAT**<br><br>**Title :** Enhancing Speech Emotion Recognition: A Focus on Energy Analysis in Six Frequency Bands with Attention Mechanism |
| **From 12h15**<br>**to 15H00** | **Lunch break** | |
| **From 15h00**<br>**to 15h40** | **Conference 3 : A. Idri** | |
| **From 15h40**<br>**to 16h10** | **Coffee break** | |
| **Parallel sessions** | | |
| | **Stochastic Calculus, Dynamical system &**<br>**control theory** | **Numerical Methods, discrete mathematics & Computer Science** |
| **From 16h10**<br>**to 18h15** | **16h10 – 16h35 : EL FATINI Mohamed**<br><br>**Title :** Stochastic modelling in epidemiology | **16h10 – 16h35 : LAKHBAB Halima**<br><br>**Title :** A hybrid approach for solving differentiable unconstrained optimization problems |
| | **16h40 – 17h : Mariem Jakhoukh**<br><br>**Title :** Null controllability for parabolic systems with dynamic boundary condition | **16h40 – 17h : Ilham EL OUARDY**<br><br>**Title :** Variational analysis of a static thermo-electro-elastic contact problem with thermal Signorini's conditions |
| | **17h05 – 17h25 : Hind El Baggari**<br><br>**Title :** Well-posedness for heat equation with inverse square potential and dynamic boundary conditions | **17h05 – 17h25 : Bidine Ez-Zobair**<br><br>**Title :** On the S-packing coloring of circulant graphs Cn(1,t) |
| | **17h30 – 17h50 : Chaouch Hicham**<br><br>**Title :** Drift parameter estimation in the Ornstein– Uhlenbeck process driven n-mixture | **17h30 – 17h50 : KHABIR Salah Eddine**<br><br>**Title :** A Genetic Algorithm Resolution for the CETSP problem |
| | **17h55 - 18h15 : Mohammed Elhachemy**<br><br>**Title :** Reflected generalized BSDE with jumps under stochastic conditions and an obstacle problem for Integral-partial differential | **17h55 - 18h15 : Hadir Nadia**<br><br>**Title :** Alzheimer disease based Artificial Intelligence diagnosis: short review and future trends |

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|---|---|---|
| | equations with non-linear Neumann boundary conditions | |

| **Thursday, June 23, 2023** | |
|---|---|
| **From 09h00 to 09h40** | **Conference 1 : O. El fallah** |
| **From 09h45 to 10h25** | **Conference 2 : J. L. da Silva** |
| **From 10h30 to 11h00** | **Coffee break** |
| **From 11h00 To11H40** | **Conference 3 : N. Igbida** |
| **From 12h00 to 14h00** | **Lunch break** |

# Abstracts : Plenary lectures

## Mean-field Doubly Reflected BSDEs: the penalization method

20 Nov
09:15
Confrence
room

Said Hamadène

Le Mans University, France

In this talk, we present the penalization method in the construction of the solution of the mean-field doubly reflected BSDEs, i.e., a quadruple of adapted stochastic processes $(Y, Z, K^{\pm})$ such that for any $t \leq T$,

$$\begin{cases} Y_t = \xi + \int_t^T f(s, Y_s, \mathbb{E}[Y_s], Z_s)ds + K_T^+ - K_t^+ - K_T^- + K_t^- - \int_t^T Z_s dB_s; \\ h(t, \omega, Y_t, \mathbb{E}[Y_t]) \leq Y_t \leq g(t, \omega, Y_t, \mathbb{E}[Y_t]); \\ \int_0^T (Y_s - h(s, Y_s, \mathbb{E}[Y_s]))dK_s^+ = \int_0^T (Y_s - g(s, Y_s, \mathbb{E}[Y_s]))dK_s^- = 0 \ (K^{\pm} \text{ are increasing processes }) \end{cases}$$

where $(f, \xi, h, g)$ are the given data of the problem. This is a joint work with Yinggu Chen and Tingshu Mu.

**References**

[1] Briand, P., Elie, R., and Hu, Y. (2018). BSDEs with mean reflection. The Annals of Applied Probability, 28(1), 482-510.

[2] Djehiche, B., Dumitrescu R. (2022). Zero-sum mean-field Dynkin games: characterization and convergence. arXiv preprint arXiv:2202.02126.

[3] Djehiche, B., Dumitrescu, R., & Zeng, J. (2021). A propagation of chaos result for a class of mean-field reflected BSDEs with jumps. arXiv preprint arXiv:2111.14315.

[4] Djehiche, Boualem and Elie, Romuald and Hamadène, Said (2019). Mean-field reflected backward stochastic differential equations. arXiv preprint arXiv:1911.06079. To appear in Annals of Applied Probability.

20 Nov
10:00
Conference
room

## Modeling and Simulation of Phase Change Problems: Parabolic and Hyperbolic Approaches

Youssef Belhamadia

The American University of Sharjah, United Arab Emirates.

Phase change problems are involved in a large number of engineering and industrial applications such as crystal growth, continuous casting, cryosurgery, ice melting, iceberg evolution, etc. An important feature of this type of problems is that the shape and position of the interface are unknown a priori and have to be determined with novel and efficient numerical methods. In recent years, a variety of numerical modeling have been developed to provide the necessary tools for understanding the physical processes. However, the numerical modeling of this type of moving interface is still extremely challenging and is an ongoing research area.

The objective of this talk is to present recent developments in mathematical modeling and numerical simulation of phase change problems. We will first derive the mathematical models for the parabolic phase change system with and without convection, which predict an infinite thermal wave speed of propagation. Then, we present a second technique in modeling these types of problems by considering an hyperbolic approach to predict the finite speed of heat propagation. Suitable numerical methods for solving the derived models using both approaches will be illustrated. Numerical simulations on water solidification, gallium melting, and continuous casting will be explored to assess the performance of the proposed techniques. A comparison with the experimental and the numerical results of the literature will be illustrated as well.

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20 Nov
15: 00
Conference
room

## Efficient valuation techniques for high dimensional dynamics

Ernst Eberlein

University of Freiburg, Germany

The increasing complexity of financial markets calls for increasingly sophisticated products. The valuation of such products often requires multidimensional dynamics. Fourier based methods represent an excellent choice for valuation due to their numerical efficiency and ease of implementation. The curse of dimensionality however can significantly hamper their applicability. We discuss in this talk potential strategies based on Monte Carlo integration as opposed to Monte Carlo simulation. The approach is illustrated by considering sophisticated insurance products, namely variable annuities.

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## Alternating Direction Method of Multiplier (ADMM): From nonlinear mechanics to data science

21 Nov
09: 00
Conference room

Jonas Koko

Clermont-Auvergne University, France.

The alternating direction method of multipliers (ADMM) is an al- gorithm for solving particular types of convex optimization problems. It was originally proposed in the mid-1970s by Glowinski & Marrocco (1975) and Gabay & Mercier (1976) for solving nonlinear partial differ- ential equations. The main idea behind the method is to separate the difficultie (e.g., linear/nonlinear, differentiable/non differentiable) by introducing an auxiliary unknown. A block Gauss-Seidel procedure is then applied to the corresponding augmented Lagrangian functional. The method takes the form of the decomposition coordination in which the coordination is ensured by the Lagrange multiplier. The algorithm was successfully applied to Stokes equation, liquid crystal problem, flow of viscoplastic fluids, displacement of flexible road, unilateral contact with or without friction, etc. ADMM is gaining a lot of popularity in data science because it often allows optimization to be done in a distributed manner, making large-size problems tractable, particluarly in statistics and machine learning. ADMM is particularly suitable for convex non differentiable norms appearing in cost functions of data science optimization prob- lems. After some convex analysis tools, we discuss applications of the ADMM algorithm to wide variety of problem from nonlinear mechanics to machine learning including least absolute values, $\ell_1$ regularization, LASSO, etc.

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## Placement et appariement de graphes pour la classification de structures

21 Nov
09: 45
Conference room

KHEDDOUCI AMACHE

Lyon 1 University, France

Placer un graphe G dans un graphe H revient à trouver une copie de G dans le graphe H. Plonger un graphe G dans un graphe H revient à trouver une injection des sommets de G sur les sommets de H de sorte que l'image d'une arête de G soit une chaine dans le graphe H. Une façon d'apparier ou de comparer G et H est de chercher un plongement de G dans H et un autre plongement de H dans G. Les deux problèmes sont liés. Dans cette présentation, on fera un tour d'horizon de certains résultats théoriques connus sur le placement de graphes, de décrire certaines dépendances entre le placement et l'appariement de graphes, et finalement de montrer comment les deux problématiques peuvent contribuer à la classification de structures d'objets les plus complexes.

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21 Nov
15 :00
Confrence
room

## Machine Learning for Medical Decision Making

Ali IDRI

Mohammed V University and Mohammed VI Polytechnic University, Moroccco

Current information and storage technologies are resulting in the explosive growth of many

business, government, and scientific databases. This has led to the development of advanced techniques and tools to assist humans extract useful information and make informed decisions from available data. Knowledge discovery in databases (KDD) has therefore become a very active research field and its applications may range from business management, and market analysis, to engineering design and medical exploration. KDD is concerned with the development of powerful and versatile tools for making sense of data. It consists of three main steps: data preprocessing (DP), Modeling, and knowledge evaluation and validation. Machine Learning is the mathematical core of KDD which deals with the application of intelligent techniques in order to obtain useful patterns, while DP deals with different real-world data imperfections for a successful use of ML techniques. In medicine, KDD can be used to extract knowledge from clinical data for effective medical diagnosis, prognosis, treatment, screening, monitoring, and management. This talk presents the findings of our recent research dealing with the use of KDD in medicine, in particular in Breast Cancer and Cardiology.

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22 Nov
09:00
Conference
room

## Dirichlet spaces and related problems

Omar El fallah

Mohammed V University, Moroccco

An operator $T$ acting on a Hilbert space $H$ is said to be a two isometry if $T^{*2}T^2 - 2T^*T + I_H = 0$, where $T^*$ denote the adjoint of $T$ and $I_H$ is the identity operator. In [1], S. Ricchter proves that an analytic cyclic two isometry can be seen as a Shift operator on some Dirichlet spaces. In this talk we will present some advances in the study of Dirichlet spaces. We will also discuss some natural problems, still open, in connection with these spaces. We will focus on the description of zero sets and on approximation problems. Estimates of the reproducing kernel and the notion of capacities associated with Dirichlet spaces will also be discussed.

**References**

[1] S. Richter. Invariant subspaces of the Dirichlet shift. Journal fur die Reine und Angewandte Mathematik, 386:205–220, 1988.

[2] O. El-Fallah, K. Kellay, J. Mashreghi, and T. Ransford. A primer on the Dirichlet space, volume 203. Cambridge University Press, 2014.

[3] O. El-Fallah, Y. Elmadani, and K. Kellay. Kernel and capacity estimates in Dirichlet spaces. Journal of Functional Analysis, 276(3):867–895, 2019.

[4] O. El-Fallah, Y. Elmadani, and I. Labghail. Extremal functions and invariant subspaces in Dirichlet spaces. Advances in Mathematics, 408:108604, 2022.

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## Green Measures for Markov Processes with Non-Local Generators with Singular Kernels

22 Nov
09:45
Conference room

José Luís da Silva

Madeira University, Portugal

In this talk we investigate the existence of Green measures for a class Markov processes associated to a non-local generators given in terms of singular kernels. Instead of Fourier analysis (used in case of non-singular kernels) we may the heat kernels bounds which allows us to show the existence of Green measures including relations between the order of singularity and the dimension.

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## Wasserstein distance vs $H^{-1}-$norm and applications in PDEs

22 Nov
11:00
Conference room

Noureddine Igbeda

Limoges University, France

The aim of this talk is to show how the Wasserstein distance and the $H^{-1}-$norm appear in the modelling of some physical phenomena in terms of PDEs. Even though both approaches produce the same models in some standard cases, we will show that this is not the case in general. In this talk, we will focus in particular on some applications in crowd motion and congestion.

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# Abstracts : Parallel Sessions

# Session 1 : Stochastic Calculus, Dynamical System and Control Theory

# Semilinear diffusion with multiplicative noise

A. Es-Sarhir
*Ibn Zohr University*

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**Abstract**

Consider a semilinear stochastic evolution equation of the type

$$dX(t) = \Big(AX(t) + B(X(t))\Big)dt + \sigma(X(t))\,dW_t, \quad t \geq 0, \qquad \text{(E)}$$

defined on a separable real Hilbert space $H$. Here $\big(A, D(A)\big)$ is a linear operator and $B$ is a nonlinear function on $H$. $(W_t)_{t\geq 0}$ is a cylindrical Wiener process in $H$ and $\sigma$ is a Nemitskii-type operator.
The equation above can be seen as an abstract formulation of many partial differential equations perturbed by random noise such as stochastic reaction diffusion, Cahn-Hilliard, and Burgers equations. The transition semigroup corresponding to (E) on the space of bounded measurable functions on $H$, $\mathcal{B}_b(H)$ is defined by

$$P_t f(x) = \mathbb{E}(f(X(t))|X(0) = x) = \int_H f(y)P(t,x,dy), \quad f \in \mathcal{B}_b(H).$$

A probability measure $\mu$ is invariant for $P_t$ if

$$\int_H P_t f(x)\,\mu(dx) = \int_H f(x)\,\mu(dx), \quad f \in B_b(H).$$

In this talk, we will go through some results concerning this semigroup. We will discuss the strong Feller property and irreducibility of $(P_t)_{t\geq 0}$. Existence and uniqueness of invariant measures will be discussed as well. demonstrate the validity of our theoretical results.

FPS

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UCA

# Pricing Americain game options in Azéma's markets: A doubly reflected BSDEs with RCLL martingales approach

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**Abstract**

In this study, we establish a fair price for an American game option traded between two insiders in a financial market driven by Azéma's martingale. To achieve this, we utilize the theory of backward stochastic differential equation driven by a right continuous with left limits (rcll) martingale with two completely separated rcll barriers in a general filtration. We prove the existence and uniqueness of a square-integrable adapted solution using the penalization method when the coefficient is stochastically Lipschitz. This solution is characterized as the fair price of the game contingent claim by applying a progressive enlargement of filtration. Moreover, we identify a saddle point for the game in the case of left upper semi-continuous obstacles.

*Keywords :* Doubly reflected backward stochastic differential equations; rcll martingales; stochastic Lipschitz coefficient; penalization method; Azéma's martingale; American game option; initial enlargement of filtration

## 1 Introduction

In contrast to American options, which only provide the buyer the choice of choosing the exercise time, American game options or game contingent claims are derivative securities introduced for the first time by Y. Kiffer [4] in the case of a perfect market model and later examined by numerous authors (e.g., [2, 3]), that allow the buyer to exercise the right to buy (call option) or sell (put option) a specific security for a specific agreed price and that permit the buyer and seller to stop them at any moment before maturity.

Valuing American game options between two insiders has long been a challenging problem in mathematical finance. This is especially true when the option is traded on the same stock of a company, as both the option seller and buyer have access to extra information beyond what is provided by the market.

The aim of this study is to provide a comprehensive overview of the use of a class of Doubly Reflected BSDEs (DRBSDEs for short) driven by a fairly rcll general martingale with two completely separated rcll obstacles in arbitrary filtered probability space under stochastic Lipschitz condition on the driver in a general filtration for valuing American game options between two insiders on the same stock of a company. We consider a market model where the dynamic of the company's stock price is driven by Azéma's martingale, and the filtration contains the natural flow of information of the public market generated by the Azéma's martingale, as well as the additional information carried by the two insiders in the sens of initial enlargement. Finally, we prove the existence of saddle points, provided there are additional regularity assumptions on the obstacles.

## 2 Doubly reflected BSDEs: Main result

The first part of this contribution aims to explore the existence and uniqueness problem, on an arbitrary filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\leq T}, \mathbb{P})$, for a class of doubly reflected backward stochastic differential equations of the following form, on an appropriate $\mathbf{L}^2$-space:

$$\begin{cases} \text{(i) } Y_t = \xi + \int_t^T f(s, Y_s, Z_s) d\langle M\rangle_s + \int_t^T dK_s^+ - \int_t^T dK_s^- - \int_t^T Z_s dM_s - \int_t^T dN_s,\ t\leq T \\ \text{(ii) } L_t \leq Y_t \leq U_t,\ 0\leq t\leq T \\ \text{(iii) If } K^{c,\pm} \text{ is the continuous part of } K^{\pm}, \text{ then } \int_0^T (Y_t - L_t) dK_t^{c,+} = 0 \\ \quad \text{and } \int_0^T (U_t - Y_t) dK_t^{c,-} = 0. \\ \quad \text{If } K^{d,\pm} \text{ is the purely discontinuous part of } K^{\pm}, \text{ then } K^{d,\pm} \text{ is } \mathcal{F}_t\text{-predictable and} \\ \quad K_t^{d,+} = \sum_{0<s\leq t} (Y_s - L_{s-})^- \text{ and } K_t^{d,-} = \sum_{0<s\leq t} (Y_s - U_{s-})^+. \end{cases} \tag{1}$$

where $(\mathcal{F}_t)_{t\leq T}$ is quasi-left continuous and satisfies the usual conditions of right-continuity and completeness and $\mathcal{F}_T = \mathcal{F}$ where $T$ is a fixed time horizon. The initial $\sigma$-field $\mathcal{F}_0$ is assumed to be trivial and $M = (M_t)_{t\leq T}$ is a real-valued square-integrable, $\mathbb{F}$-martingale. It is presumed that M is an rcll process because the filtration $\mathbb{F}$ is right-continuous and an rcll modification of any $\mathbb{F}$-martingale is known to exist.

The driver $f$ is assumed to satisfy th so called *stochastic Lipschitz* condition and the barriers $(L_t)_{t\leq\tau}$ and $(U_t)_{t\leq T}$ are real-valued $\mathcal{F}_t$-progressively measurable rcll processes satisfying: $L_T \leq \xi \leq U_t$, $L_t < U_t$ and $L_{t-} < U_{t-}$, $\mathbf{P}$-a.s. As we have already mentioned, our objective is to prove the following:

**Theorem 2.1.** *Under some suitable assumption on* $(\xi, f, L, U)$*, the DRBSDE* (1) *admits a unique solution* $\left(Y_t, Z_t, K_t^-, K_t^+, N_t\right)_{t\leq T}$.

## 3 American game Option in Azéma's markets

Let $\mathbb{G} = (\mathcal{G}_t)_{t\leq T}$ is the one generated by the Azéma's martingale $(M_t)_{t\leq T}$ made right-continuous and complete, characterized by the so-called *structure equation*

$$d[M,M]_t = dt - M_{t-} dM_t. \tag{2}$$

The second objective of this study is to investigate the cost problem of an American game option between two insiders in a financial market governed by the dynamics of the Azéma martingale equation (2). The market is described by the following equations:

$$\begin{cases} dS_t^0 = r_t S_t^0 dt, \quad S_0^0 = 1, \\ dS_t = S_{t-} dM_t, \quad S_0 = 1. \end{cases} \tag{3}$$

Here, $(r_t)_{t\leq T}$ is a positive process that represents the interest rate. In this area, there has been limited research conducted. However, a paper by Dritschel and Protter [1] suggests substituting Azéma martingales for Brownian motion in the financial market model. On the other hand, our mathematical model assumes that at time $t=0$, both buyers and sellers have access to the public information $\mathbb{G}$, as well as two $\mathcal{F}$-measurable random variables, $X_1$ and $X_2$. Therefore, to apply the standard results, we use the associated right and quasi-left continuous filtration, denoted by $\mathbb{F}=(\mathcal{F}_t)_{t\leq T}$:

$$\mathcal{F}_t=\bigcap_{\varepsilon>0}\{\mathcal{G}_{t+\varepsilon}\vee\sigma(X_1)\vee\sigma(X_2)\},\quad t\in[0,T],$$

completed by all the $\mathbb{P}$-null sets of $\mathcal{F}$. This is known as the *initial enlargement* of the filtration $\mathbb{G}$ by the random variables $X_1$ and $X_2$.

Our objective here is to use the doubly reflected BSDEs to determine the fair price of this game option, which refers to the amount of money paid by the buyer at time $t=0$.

Finally, we prove the existence of saddle points, provided there are additional regularity assumptions on the obstacles.

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# Sufficient and Necessary Conditions for Stochastic Near-Optimal Control in a Within-Host Infectious Disease Model.

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**Abstract**

This paper is concerned with necessary as well as sufficient conditions for near- optimality for drug therapy in a stochastic viral model incorporating stochastic fluctuations and combining the lytic and nonlytic immune responses where the system is governed by stochastic differential equations (SDE's). According to the adjoint equations and its solution , we estimate the error bound for the near optimality. Then, using Ekeland's variational principle and some stability results on the state and adjoint processes, with respect to the control variable, we will prove sufficient and necessary conditions to minimize the cost function. Using control treatment, numerical illustrations are introduced to compare with theoretical.

*Keywords :* Random viral model, control treatment, near- optimality, adjoint equation, Ekeland's variational principle .

## 1 Introduction

The development of optimal disease intervention techniques using optimal control theory [3] has shown to be a helpful way of comprehending how to stop the spread of infectious illnesses. The strategy involves reducing the expense of infection, the expense of applying the control.

Recent years have seen a significant increase in interest in stochastic near-optimal control [4] for managing dynamics in a variety of practical domains, including epidemiology ,oncology [1], and finance [2] in the reason that near-optimal controls always exist and it is usually much easier to obtain near optimal controls than optimal ones, both analytically and numerically

On the other hand , many mathematical models have been proposed to describe, analyze and control the viral behavior within the host individual with and without immune responses, where the immune system is a complex,which plays a prominent function during the detection of a strange substance to the body and inhibits the development of contamination. For this reason, mathematical models were applied to identify the principles underlying the interactions between viruses and immunological effector systems and interpret existing empirical data to address experimental data on virus infection in mice deficient in lytic or nonlytic immune effector mechanisms

In this work, incorporating stochastic fluctuations, we consider a viral infection model to describe the role of lytic and nonlytic immune responses. Lytic immunity is defined as the destruction of

# A class of Gaussian Volterra processes as extensions of the fractional Brownian motion

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**Abstract**

We consider the Gaussian Volterra process $X^\theta = \{X^\theta(t),\ t \in [0,T]\}$, $\theta = (\alpha,\beta,\gamma)$ recently defined in [2] as

$$\begin{aligned} X^\theta(t) &= \int_0^t K^\theta(t,s)dB(s), t \geq 0,\ \ \theta = (\alpha,\beta,\gamma) \\ \text{with}\ \ K^\theta(t,s) &= s^\alpha \int_s^t u^\beta (u-s)^\gamma du \mathbf{1}_{(0<s\leq t)}, \end{aligned}$$

where $\{B(s),\ 0 \leq s \leq T\}$ is Wiener process and $\alpha > -1/2,\ \gamma > -1,\ \alpha+\beta+\gamma > -3/2$. Here we specify the parameters $\theta$ for which $X^\theta$ is non Markovian, semimartingale, and exhibits long-range dependence. Finally, by using its Paley-Wiener-Zygmund representation we establish its continuity in $\theta$, uniformly in $t$. It is worth to mention that $X^\theta$ reduces to the generalized Riemann-Liouville fractional Brownian motion (**fBm**) [1] when $\beta = 0$, while the fBm is retrieved in the case $\alpha = 1/2 - H$, $\beta = H - 1/2$, $\gamma = H - 3/2$ with $H \in (1/2, 1)$.

*Keywords :* Long-range dependence; semimartingale property; non Markovian process; Paley-Wiener-Zygmund representation

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# Geometric approach of a product form stationary distribution for an SRBM in three dimensions

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**Abstract**

We focus on the product form of a three-dimensional Semimartingale Reflecting Brownian Motion (SRBM) on a nonnegative orthant. Assuming that SRBM is positive recurrent and the stationary distribution exists. We describe the SRBM data $(\Sigma, \mu, R)$ by the geometric objects, and we provide a geometric condition that characterizes the existence of product form stationary distribution.

*Keywords :* SRBM, Stationary distribution, positive recurrent, product form.

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## 1 Introduction

We consider a Semi-martingale Reflecting Brownian Motion (SRBM) lives on the state space $\mathbf{R}^3_+$, the data of this process are a covariance matrix $\Sigma$, a drift vector $\mu$, and a reflection matrix $R$ satisfying the positive recurrence conditions.
We define the geometric interpretation of the data of SRBM: an ellipsoid that is specified by $(\Sigma, \mu)$, and three plans that are specified by R. We prove in the theorem that the SRBM has a product form stationary distribution if and only if R is an admissible matrix and $\theta^{ij(.,r)}$ "the symmetry point" of the intersection of the ellipsoid and plans are equal.

## 2 Geometrical interpretation

We assume that the SRBM has a stationary distribution, and it satisfies the positive recurrence conditions ( see, [1] and [3]).

**Theorem 2.1.** *The three-dimensional $(\Sigma, \mu, R)$ SRBM has a product form stationary distribution if and only if*

- *R is an admissible matrix,*
- $\theta^{ij(i,r)} = \theta^{ij(j,r)}, \quad i \neq j$

*Proof.*
- Let the three-dimensional polynomials:

$$\gamma(\theta) = -\frac{1}{2} < \theta, \Sigma\theta > - < \mu, \theta >, \theta \in \mathbf{R}^3$$

$$\gamma_i(\theta) = < R^i, \theta >, i \in \{1,2,3\}$$

Where $R^i$ is the ith column of the reflection matrix. those polynomials uniquely determine the primitive data of SRBM.

- We characterize the product form condition of the three-dimensional SRBM through the two-dimensional SRBM,
- It follows that the two-dimensional SRBM has a product form stationary distribution from the theorem (5.1)(see, [2]).

□

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# Optimal control of a contact problem in Orlicz spaces

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**Abstract**

In this work, we study a static contact problem with a non-polynomial growth of the elasticity. The contact is frictionless with normal compliance. We prove the existence and uniqueness results for its Weak solution in reflexive Orlicz-Sobolev space. We state the optimal contact and prove that it has at least one solution.

*Keywords :* Orlicz space, Elastic material; Frictional contact problem; Weak solution; Optimal control

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# Structure of positive radial solutions of a nonlinear boundary value problem including the p-Laplacian operator

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**Abstract**

In this work, we study the following nonlinear boundary value problem

$$(P)\begin{cases}(|u'|^{p-2}u')' + \frac{N-1}{r}|u'|^{p-2}u' + \alpha u(r) + \beta r u'(r) + |u|^{q-1}u(r) = 0, & r > 0\\ u(0) = A > 0, \quad u'(0) = 0, \end{cases}$$

where $p > 2$, $q > 1$, $N \geq 1$, $\alpha > 0$ and $\beta > 0$.

We show existence and uniqueness of solutions of problem $(P)$ and we give their classification. Moreover, we establish under some appropriate assumptions that the positive solution has the following behavior near infinity

$$\lim_{r\to+\infty} r^{\frac{\alpha}{\beta}} u(r) = \Gamma$$

and

$$\lim_{r\to+\infty} r^{\frac{\alpha}{\beta}+1} u'(r) = \frac{-\alpha}{\beta}\Gamma,$$

where $\Gamma$ is a positive constant that depends on $N$, $p$, $q$, $\alpha$ and $\beta$.

*Keywords :* Global existence, Asymptotic behavior, Energy function, Radial solution.

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# LU Decomposition Method to Solve Intuitionistic Fuzzy Linear Systems

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**Abstract**

Systems of linear equations play an essential role in several areas such as physics, mathematics and engineering. Usually, in many real world problems, we deal with imprecise data. Therefore, some parameters are presented as intuitionistic fuzzy number rather than crisp number.
The purpose of this paper is to solve the intuitionistic fuzzy linear system, with crisp coefficients matrix and intuitionistic fuzzy right hand side, using LU decomposition. The method is discussed, then considered in a case when the matrix is symmetric positive definite and finally illustrated by numerical examples.
*Keywords :* Intuitionistic fuzzy number, intuitionistic fuzzy linear system, LU decomposition.

[3], [1], [2], [4], [5].

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# SOLVING HIGHER ORDER INTUITIONISTIC FUZZY DIFFERENTIAL EQUATIONS

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**Abstract**

In this paper, We provide an existence and uniqueness result for the second-order intuitionistic fuzzy differential equation satisfying a lipschitz condition, For this problem using the fixed point theorem and an example is provided to illustrate the result.

*Keywords :*
(1) Intuitionistic fuzzy solution.
(2) Intuitionistic fuzzy initial value problem.
(3) Fixed Point.

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# Null approximate impulse controllability for parabolic degenerate singular equations via logarithmic convexity

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**Abstract**

The purpose of this work is to investigate the null approximate controllability with an impulsive control of the following one-dimensional degenerate singular system $u_t - (au_x)_x - \frac{\mu}{x^\beta}u = 0,\ x \in (0,1)$, where the diffusion coefficient $a(\cdot)$ is degenerate at $x = 0$, the parameters $\beta \geq 0\, and\, \mu \in \mathbb{R}$ satisfy suitable assumptions. To this aim, we derive a logarithmic convexity estimate for the solution of the above system by using a Carleman commutator approach.

*Keywords :* Impulsive approximate controllability, impulsive control problems,logarithmic convexity, Carleman commutator.

## 1 Introduction

Over the last several years, important process has been made in the null controllability for parabolic equations. After the pioneering works [1,3,5,7], there has been substantial progress in understanding the controllability properties of degenerate singular parabolic equations. In particular, the authors in [2] have provided a full analysis of the equation $\partial_t u - (au_x)_x - \frac{\mu}{x^\beta}u = 0,\ (x,t) \in (0,1)\times(0,T)$, such that the degenerate diffusion coefficient $a$ satisfies the following assumption

**Hypothesis 1.** *We suppose that the diffusion coefficient $a(\cdot)$ satisfies the following hypothesis*

- *The weakly degenerate case (WD):*
  1. $a \in \mathcal{C}([0,1]) \cap \mathcal{C}^1((0,1]),\ a(0) = 0\, and\, a > 0\, in\ (0,1]$;
  2. $\exists K_a \in [0,1)\ such\, that\, xa'(x) \leq K_a a(x),\ \forall x \in [0,1]$.
- *The strongly degenerate case (SP):*
  1. $a \in \mathcal{C}^1([0,1]),\ a(0) = 0\, and\, a > 0\, in\ (0,1]$;
  2. $\exists K_a \in [1,2)\ such\, that\, xa'(x) \leq K_a a(x),\ \forall x \in [0,1]$;
  3. $\begin{cases} \exists \theta \in (1, K_a],\ x \mapsto \dfrac{a(x)}{x^\theta}\ is\, nondecreasing\, near\, 0\, if\ K_a > 1, \\ \exists \theta \in (0,1),\ x \mapsto \dfrac{a(x)}{x^\theta}\ is\, nondecreasing\, near\, 0\, if\ K_a = 1. \end{cases}$

where $K_a$ represents the degree of degeneracy of the function $a$ at $x = 0$. In the present reference, the authors have shown that the singular degenerate equation is null controllable by proving a Carleman inequality of the associated adjoint problem with suitable conditions on $\beta$ and $\mu$. On the other hand, the approximate controllability of parabolic equations with impulse control starts to gain more attention recently, one can mention many works in this field such as [4,6]. This type of control is very weak since it only acts in a subdomain at one instant of time, which makes the controllability problem more challenging. The aim of this work is to study the approximate controllability of singular degenerate parabolic equation with impulse control by using the strategy in [4] which is based on a logarithmic convexity estimate obtained by a Carleman commutator approach.

## 2 Main results

Let us first assume that the function $a$ satisfies Hypothesis 1 and one of the following assumptions

- sub-critical potentials:

$$\begin{aligned} &K_a \in [0,2[, \;\; 0 < \beta < 2 - K_a \;\; \textit{and} \;\; \mu \in \mathbb{R}; \\ &K_a \in [0,2[\setminus\{1\}, \;\; \beta = 2 - K_a \;\; \textit{and} \;\; \mu < \mu^*(a, K_a). \end{aligned} \tag{1}$$

- critical potentials:

$$K_a \in [0,2[\setminus\{1\}, \;\; \beta = 2 - K_a \;\; \textit{and} \;\; \mu = \mu^*(a, K_a), \tag{2}$$

where $\mu^*(a, K_a)$ is the optimal constant of the Hardy-type inequality proved in [2]. Now, let $\omega$ be a nonempty open subset of $(0,1)$ and $T > 0$. Consider the following impulse degenerate singular system

$$\begin{cases} \partial_t y - (a y_x)_x - \dfrac{\mu}{x^\beta} y = 0, & \text{in } (0,1) \times (0,T) \setminus \{\tau\}, \\ y(\cdot,\tau) = y\left(\cdot,\tau^-\right) + \mathbb{1}_\omega h(\cdot,\tau), & \text{in } (0,1), \\ y(1,t) = 0, & \\ \begin{cases} y(0,t) = 0, & (WD), \\ (a y_x)(0,t) = 0, & (SD), \end{cases} & \text{on } (0,T), \\ y(0,x) = y_0(x), & \text{on } (0,1), \end{cases} \tag{3}$$

where $\tau \in (0,T)$ is an impulse time, $y_0 \in L^2(0,1)$ is the initial data, $y(.,\tau^-)$ denotes the left limit of the function $y$ at time $\tau$, $\mathbb{1}_\omega$ is the characteristic function of $\omega$ and $h(\cdot,\tau) \in L^2(\omega)$ is the impulse control. The main of this work is to prove that the impulse system 3 is approximately null controllable, that is,

**Definition 2.1.** *The system 3 is said to be approximately null impulse controllable at time $T$ if for any $\varepsilon > 0$ and $y_0 \in L^2(0,1)$, there exists a control $h(\cdot,\tau) \in L^2(\omega)$ such that the following estimate holds*

$$\| y(\cdot,T) \|_{L^2(0,1)} \leq \varepsilon \| y_0 \|_{L^2(0,1)} . \tag{4}$$

*This leads for any $\varepsilon > 0$ and $u_0 \in L^2(0,1)$ to the definition of the cost of null approximate impulse control at time $T$*

$$L(T,\varepsilon) := \sup_{\|y_0\|=1} \inf_{h(\cdot,\tau) \in \mathcal{R}_{T,\varepsilon,y_0}} \| h \|_{L^2(\omega)}$$

*with $\mathcal{R}_{T,\varepsilon,y_0} := \left\{ h(\cdot,\tau) \in L^2(\omega) : \text{the solution } y \text{ of system } (3) \text{ satisfies } \| y(\cdot,T) \| \leq \varepsilon \| y_0 \| \right\}$.*

Now, we give the main result of the null approximate impulse controllability

**Theorem 2.2.** *Assume that Hypothesis 1 and* (1) *or* (2) *hold. Then, the system* (3) *is null approximate impulse controllable at any time* $T > 0$*. Moreover, for any* $\varepsilon > 0$*, there exist some positive constants* $C_1, C_2, \kappa$ *and* $\delta$ *such that the cost of null approximate impulse control function at time* $T$ *satisfies*

$$L(T,\varepsilon) \leq C_1 \frac{e^{C_2\left(T+\frac{1}{T}\right)+\kappa\tau}}{\varepsilon^{\delta}}. \tag{5}$$

Therefore, in order to prove the above result we need to provide an observabilition estimate for every solution of the following non-impulsive system

$$\begin{cases} \partial_t u - (au_x)_x - \dfrac{\mu}{x^{\beta}} u = 0, & \text{in } Q := (0,1)\times(0,T), \\ u(1,t) = 0, & \\ \begin{cases} u(0,t) = 0, & (WD), \\ (au_x)(0,t) = 0, & (SD), \end{cases} & \text{on } (0,T), \\ u(0,x) = u_0(x), & \text{on } (0,1). \end{cases} \tag{6}$$

Thus, for any solution $u$ of 6 associated to $u_0$ we obtain the following Lemma

**Lemma 2.3.** *Assume that Hypothesis 1 and 1-2 hold true. Let* $\omega$ *be a sub-interval of* $(0,1)$*. Then there exist positive constants* $C_1, C_2, C_3$ *and* $\rho \in (0,1)$ *such that the following observability estimate is satisfied for every solution u of 6*

$$\| u(\cdot,T) \| \leq \left( C_1 e^{C_2\left(\frac{1}{T}+T\right)} \| u(.,T) \|_{L^2(\omega)} \right)^{\rho} \| u(\cdot,0) \|^{1-\rho}. \tag{7}$$

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# Backward Stochastic Evolution Inclusions in UMD Banach Spaces

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**Abstract**

In this talk, we discuss the existence of a mild $L^p$-solution for the backward stochastic evolution inclusion (BSEI for short) of the form

$$\begin{cases} dY_t + AY_t dt \in G(t, Y_t, Z_t)dt + Z_t dW_t, & t \in [0, T] \\ Y_T = \xi, \end{cases}$$

where $W = (W_t)_{t \in [0,T]}$ is a standard Brownian motion, $A$ is the generator of a $C_0$-semigroup on a UMD Banach space $E$, $\xi$ is a terminal condition from $L^p(\Omega, \mathcal{F}_T; E)$, with $p > 1$ and $G$ is a set-valued function satisfying some suitable conditions.

The case when the processes with values in spaces that have martingale type 2, has been also studied.

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# Stochastic modelling in epidemiology.

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**Abstract**

There are many cases when deterministic models are not adequate. For example, dynamics fluctuations are not smoothed out by statistical averaging, and the time evolutions of such systems are therefore stochastic. The randomness in the system usually cannot be ignored, thus, one is forced to adopt a stochastic description. The stochastic models take into account in addition of the mean trend, the variance structure around it. In this work we are interested in the study of the behavior of the global positive solution for an epidemic model characterized by temporary immunity. We analyze the qualitative behavior of the disease around both the disease-free and endemic equilibriums. We show that the solution does random fluctuations with an intensity related to the values of the volatility or jump increments.

*Keywords :* Stochastic epidemic model, Extinction, Persistence.

## 1 Introduction

COVID-19 is a novel infectious viral disease caused by the SARS-CoV-2 virus and has been declared a global pandemic by the World Health Organization [2].In December 2019, the COVID-19 was first discovered in Wuhan and caused the first pandemic in the world. The virus is primarily transmitted human-to-human via oral ,coughing, sneezing, where the virus-contaminated environment play a lesser role in the propagation of disease appears to be transferred mostly through peoples interaction in close proximity.

Therefore the only way to curb the spread of this coronavirus is to isolate the initially infected population or the vaccination as showed by guide line of World Health Organization. In June of 2020, the COVID-19 virus has infected more than 10,927,025 people and 521,512 deaths in all over the world [1] .

The aim of mathematical models in epidemics in general is to describe the spread of a particular disease in the best possible way, then the coronavirus COVID-19 has gained a big interest from many researchers to deepen understanding and grasping the valuable inferences through mathematical modeling [3, 4], this type of modeling is then divided into different types. Among them the best is the stochastic modeling approach because it gives valuable results On comparison of the deterministic approach since the environment varies randomly. In reality, the environment varies randomly, The environmental perturbation can involve a number of factors such as health habits, medical quality , which may affect the others factors ( birth rate, death rate, etc.) In particular, for human infectious diseases, the the spread of the epidemic is random due to the unpredictability of person-to-person contact. That was the motivation for the transition from deterministic models to their stochastic counterparts.

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# NULL CONTROLLABILITY FOR PARABOLIC SYSTEMS WITH DYNAMIC BOUNDARY CONDITION

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**Abstract**

In this paper, we study the null controllability of systems of ncoupled parabolic equations with dynamic boundary conditions, where the coupling and control matrices A and B are constant in time and space. Being different to the case of static boundary conditions, we will show that the Kalman rank condition $rank[B, AB, \dots, A^{n-1}B] = n$ is a sufficient condition, we also show that it is necessary for the null controlability under an extra assumption. The null controlability result will be proved by proving Carleman and observability inequalities for the corresponding adjoint problem.

*Keywords :* Parabolic systems, coupled systems, dynamic boundary conditions, Carleman estimate, null controllability, observability, Kalman condition

## 1 Introduction

The null controllability of parabolic equations with internal and boundary controls has attracted a lot of interest. The common tool in most of previous works is the development of suitable Carleman estimates of the corresponding adjoint problems and their observability inequalities, see e.g., [2, 8, 9, 11]. Recently, intensive interesting results are obtained for systems of $n$ coupled parabolic equations, when the coupling and control matrices even depend on time. In [1] and [10], the authors considered the case of $n$ coupled cascade systems with $r$ control forces. Ammar-Khodja and his collaborators in [3–7], have considered the general full $n$ coupled systems. They characterized the null controllability of these systems in terms of the Kalman rank condition $rank[A|B] = n$. Note that all these mentioned results are obtained for Dirichlet and for inhomogeneous or nonlinear Neumann boundary conditions.

In this paper, we characterize the null controllability of $n$-coupled linear parabolic equations with dynamic boundary conditions of surface diffusion type, via $m$ control forces:

$$\begin{cases} \partial_t y - d\Delta y + Ay = f + B\mathbf{1}_\omega v(t,x) & \text{in } \Omega_T, \\ \partial_t y_\Gamma - \delta\Delta_\Gamma y_\Gamma + d\partial_\nu y + A_\Gamma(t,x) y_\Gamma = g & \text{on } \Gamma_T, \\ (y, y_\Gamma)|_{t=0} = (y_0, y_{0,\Gamma}) & \text{in } \Omega \times \Gamma, \end{cases} \tag{1}$$

where $A = (a_{ij})_{1\le i,j\le n}$, $A_\Gamma(t,x) = (a^\Gamma_{ij}(t,x))_{1\le i,j\le n}$ are matrices, $B$ is a $n \times m$ matrix, $y = (y_1, \cdots, y_n)^*$, $y_\Gamma = (y_{1,\Gamma}, \cdots, y_{n,\Gamma})^*$, $v = (v_1, \cdots, v_m)^*$, $f = (f_1, \cdots, f_n)^*$ and $g = (g_1, \cdots, g_n)^*$.

In this paper, for constant coupling and control matrices $A$ and $B$, we show that the full coupled parabolic system (1) is null controllable by means of $m$ control forces if the Kalman rank condition is satisfied

$$rank[B, AB, \ldots, A^{n-1}B] = n. \tag{2}$$

## 2 Carleman estimates

In this section, we show a Carleman estimate for the following adjoint problem

$$\begin{cases} -\partial_t \varphi - d\Delta\varphi + A^*\varphi = f(t,x) & \text{in } \Omega_T, \\ -\partial_t \varphi_\Gamma - \delta\Delta_\Gamma \varphi_\Gamma + d\partial_\nu \varphi + A_\Gamma^*(t)\varphi_\Gamma = g(t,x) & \text{on } \Gamma_T, \\ (\varphi, \varphi_\Gamma)|_{t=T} = (\varphi_0, \varphi_{0,\Gamma}) & \text{in } \Omega \times \Gamma. \end{cases} \tag{3}$$

The main result of this section is the following Carleman estimate.

**Theorem 2.1.** *Let $T > 0$, $\omega \Subset \Omega$ be an open nonempty subset. Let $A$ and $B$ be the matrices from (3) and satisfy the condition (2), and $A_\Gamma \in L^\infty(\Gamma_T, \mathcal{L}(\mathbb{R}^n))$. Define $\eta^0$, $\alpha$ and $\gamma$ as above with respect to $\omega$. Then, there exists $\hat{\lambda} > 0$, $l \geq 3$ and $l^1 \geq 0$ such that for every $\lambda \geq \hat{\lambda}$, we can choose positive constants $s_0(\lambda, l)$ and $C = C(\lambda, l)$ such that every solution $(\varphi, \varphi_\Gamma)$ of (3) satisfies*

$$\sum_{i=1}^{n} J(0,\varphi_i) \leq C\left(\int_{\omega\times(0,T)} s^l \gamma^l e^{-2s\alpha}|B^*\varphi|^2\,dx\,dt + s^{l^1}\int_{\Omega_T} e^{-2s\alpha}\gamma^{l^1}|f|^2\,dx\,dt + \int_{\Gamma_T} e^{-2s\alpha}|g|^2\,dS\,dt\right) \tag{4}$$

*for all $s \geq s_0(\lambda, l)$. The term $J(k,z)$ is given by*

$$\begin{aligned} J(k,z) =& s^{k+1}\int_Q \gamma^{k+1}e^{-2s\alpha}|\nabla z|^2\,dx\,dt + s^{k+1}\int_{\Gamma_T}\gamma^{k+1}e^{-2s\alpha}|\nabla_\Gamma z|^2\,dS\,dt \\ &+ s^{k+3}\int_{\Omega_T}\gamma^{k+3}e^{-2s\alpha}|z|^2\,dx\,dt + s^{k+3}\int_{\Gamma_T}\gamma^{k+3}e^{-2s\alpha}|z|^2\,dS\,dt \\ &+ s^{k+1}\int_{\Gamma_T}\gamma^{k+1}e^{-2s\alpha}|\partial_\nu z|^2\,dS\,dt. \end{aligned}$$

## 3 Null controllability

To show that the Kalman condition (2) is a sufficient condition for the null controllability of system (5), we use the Carleman estimate (4) to show an equivalent result which consists in deriving an observability inequality for the backward system

$$\begin{cases} \partial_t y - d\Delta y + Ay = B\mathbf{1}_\omega v(t,x) & \text{in } \Omega_T, \\ \partial_t y_\Gamma - \delta\Delta_\Gamma y_\Gamma + d\partial_\nu y + A_\Gamma(t)y_\Gamma = 0 & \text{on } \Gamma_T, \\ (y, y_\Gamma)|_{t=0} = (y_0, y_{0,\Gamma}) & \text{in } \Omega \times \Gamma, \end{cases} \tag{5}$$

where $A \in \mathcal{L}(\mathbb{R}^n)$, $A_\Gamma(\cdot) \in L^\infty(\Gamma_T, \mathcal{L}(\mathbb{R}^n))$, $B \in \mathcal{L}(\mathbb{R}^m, \mathbb{R}^n)$ and $v \in L^2(\omega_T, \mathbb{R}^m)$. Now, we state and show the observability inequality result.

**Proposition 3.1.** *Let $T > 0$, $\omega \Subset \Omega$ be nonempty and open, $d, \delta > 0$, $A \in \mathcal{L}(\mathbb{R}^n), B \in \mathcal{L}(\mathbb{R}^m, \mathbb{R}^n)$ such that (2) holds. There is a constant $C > 0$ such that for all $\varphi_T \in \mathbb{L}^2$ the mild solution $\varphi$ of the backward problem (3) withf $= g = 0$ satisfies*

$$\|\varphi(0,\cdot)\|_{\mathbb{L}^2}^2 \leq C\int_{\omega_T}|B^*\varphi|^2\,dx\,dt. \tag{6}$$

**Theorem 3.2.** *Let A and B such that rank*$[A|B] = l < n$*, and* $X := \mathrm{Im}[A|B]$*, and assume that* $A_\Gamma X \subset X$*. Then, for every* $(y_0, y_{0,\Gamma}) \in \mathbb{L}^2$ *there is a control* $v \in L^2(\omega_T, \mathbb{R}^m)$ *such that the solution* $(y, y_\Gamma)$ *for system* (5) *satisfies*

$$y_i(T,\cdot) = 0 \quad in \ \Omega, \quad y_{i,\Gamma}(T,\cdot) = 0 \quad on \ \Gamma, \ 1 \leq i \leq n$$

*if and only if* $(y_0, y_{0,\Gamma}) \in L^2(\Omega, X) \times L^2(\Gamma, X)$.

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# Well-posedness for heat equation with inverse square potential and dynamic boundary conditions

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**Abstract**

We start by proving the well-posedness for the heat equation with an inverse square potential subject to dynamic boundary conditions on a $C^2$ bounded domain contains the origin, then we study the properties of the semigroup : compacteness and positivity.

*Keywords :* Singular heat equation, boundary conditions, compactness, Hardy-Poincaré inequality

## 1 Introduction

We study the well-posedness for linear heat equations with singular potentials and dynamic boundary conditions in bounded domain. More precisely, we focus on the so-called inverse-square potential of the form $\frac{\mu}{|x|^2}$ arising, for example, in the context of combustion theory [ [3], [5], [6]]. Let $T > 0$ be a fixed final time and let $\Omega \subset \mathbb{R}^n$ $(n \geq 3)$ be a bounded domain such that $0 \in \Omega$ with smooth boundary $\Gamma = \partial\Omega$ of class $C^2$. We denote by $\Omega_T = (0,T) \times \Omega$ and $\Gamma_T = (0,T) \times \Gamma$. Let $0 \leq \mu < \mu^*(n) := \frac{(n-2)^2}{4}$. We consider the following heat equation with a singular potential subject to dynamic boundary conditions

$$\begin{cases} \partial_t y - \Delta y - \dfrac{\mu}{|x|^2} y = f, & \text{in } \Omega_T, \\ \partial_t y_\Gamma - \Delta_\Gamma y_\Gamma + \partial_\nu y = g, & \text{on } \Gamma_T, \\ y_\Gamma(t,x) = y_{|\Gamma}(t,x), & \text{on } \Gamma_T, \\ (y, y_\Gamma)|_{t=0} = (y_0, y_{0,\Gamma}), & \Omega \times \Gamma, \end{cases} \tag{1}$$

Where $(f,g) \in L^2(\Omega, \dot{x}) \times L^2(\Gamma, \dot{S})$. The Laplace operator is denoted by $\Delta = \Delta_x$. The trace of $y$ is $y_{|\Gamma}$, and the normal derivative is $\partial_\nu y := (\nabla y \cdot \nu)_{|\Gamma}$, where $\nu(x)$ is the unit outward normal at $x \in \Gamma$. Let g be the standard Riemannian metric on $\Gamma$ induced by $\mathbb{R}^N$.
The heat equation with inverse square potential and static boundary conditions has been well studied in the literature. The wellposedness and blow-up phenomena of this kind of singular equation have been first studied in [2, 7, 8].

## 2 Footnotes, Verbatim, and Citations

The system (1) can be written as a Cauchy problem

$$\text{(ACP)}\begin{cases}\partial_t Y = \mathcal{A}_\mu Y + F, & 0 < t < T,\\ Y(0) = Y_0,\end{cases} \tag{2}$$

where $Y_0 := (y_0, y_{0,\Gamma})$, $F = (f, g)$ and the linear operator $\mathcal{A}_\mu\colon D(\mathcal{A}) \longrightarrow \mathbb{L}^2$ given by

$$\mathcal{A}_\mu = \begin{pmatrix}\Delta + \frac{\mu}{|x|^2} & 0\\ -\partial_\nu & \Delta_\Gamma\end{pmatrix},\ D(\mathcal{A}_\mu) = \left\{(y, y_\Gamma) \in \mathbb{H}^1 : \Delta y + \frac{\mu}{|\,x\,|^2} y \in L^2(\Omega) \text{ and } \Delta_\Gamma y_\Gamma - \partial_\nu y \in L^2(\Gamma)\right\}. \tag{3}$$

**Theorem 2.1.** *The operator $\mathcal{A}_\mu$ generates an analytic $C_0$-semigroup on $L^2(\Omega) \times L^2(\Gamma)$.*

Let $(e^{t\mathcal{A}_\mu})_{t \geq 0}$ be the semigroup generated by $\mathcal{A}_\mu$. Our main point is proving the following result

**Theorem 2.2.** *The semigroup $(e^{t\mathcal{A}_\mu})_{t \geq 0}$ is compact for all $t > 0$.*

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# Drift parameter estimation in the Ornstein– Uhlenbeck process driven $n$-mixture

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**Abstract**

This paper focuses on the problem of estimating the drift parameter of the Ornstein-Uhlenbeck process, which is used to describe a range of phenomena in physics, finance, and other fields.
Specifically, we consider a SDE given by $dX_t = \lambda X_t dt + \sum_{k=1}^{n} \alpha_k dB_t^{H_k}$, $t \geq 0$ with an unknown parameter $\lambda > 0$ and $\left(B^{H_k}\right)_{k=1}^{n}$ are independent fBm's with different Hurst index $H_k \in (0,1)$. We propose an estimator, $\widetilde{\lambda}_t$ of $\lambda$ , based on observations $\{X_s,\ s \in [0,t]\}$, and establish both strong consistency and asymptotic distribution of our estimator $\widetilde{\lambda}_t$ when $t \to \infty$ .

*Keywords :* fractional Brownian motion, Gaussian processes, long-range dependence, multi-mixed fractional Brownian motion, multi-mixed fractional Ornstein–Uhlenbeck process, stationary processes.

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# Reflected generalized BSDE with jumps under stochastic conditions and an obstacle problem for Integral-partial differential equations with non-linear Neumann boundary conditions

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**Abstract**

By a probabilistic approach, we look at an obstacle problem with non-linear Neumann boundary conditions for parabolic semi-linear Integral-partial differential equations (IPDEs). We prove the existence of a continuous viscosity solution of this problem. The non-linear part of the equation and the Neumann condition satisfy the stochastic monotonicity condition on the solution variable. Furthermore, the non-linear part is stochastic Lipschitz on the parts depending on the gradient and the integral of the solution. It should be noted that the existence of the viscosity solution for this problem has recently been investigated in [1] using a standard monotonicity and Lipschitz conditions. In this paper, we show that the solution of the related reflected generalized backward stochastic differential equations (BSDEs) with jumps exists and is unique when the barrier is càdlàg (RCLL) and the generators satisfy stochastic monotonicity and Lipschitz conditions. In this case, we get a comparison result.

*Keywords :* IPDE, Viscosity solution, Obstacle, Generalized BSDE with jumps, Reflected BSDE, Stochastic monotone, Stochastic Lipschitz.

## 1 Introduction

Let $G$ be an open connected bounded domain of $\mathbb{R}^d$ $(d \geq 1)$ which is such that for a function $\Phi \in C_b^2(\mathbb{R})$, $G$ and its boundary $\partial G$ are characterized by $G = \{\Phi > 0\}$, $\partial G = \{\Phi = 0\}$ and for any $x \in \partial G$, $\nabla\Phi(x)$ the unit normal vector pointing toward the interior of $G$.
Let us consider the following obstacle problem of parabolic Integral-Partial Differential Equation with nonlinear Neumann Boundary conditions, by suppressing the dependence on $(t,x)$ :

$$\begin{cases} (u-\ell) \wedge \Big\{(u-h) \vee \big[-\dfrac{\partial u}{\partial t} - \mathcal{L}u - f\big(t,x,u,(\nabla u\sigma),\mathcal{B}u\big)\big]\Big\} = 0, \quad \forall (t,x) \in [0,T]\times G; \\ u(T,x) = H(x), \quad \forall x \in G; \\ \dfrac{\partial u}{\partial n} + g(t,x,u) = 0, \qquad \forall x \in \partial G \end{cases} \tag{1}$$

where

- $\mathcal{L} = R + S$ is the second-order integral-differential operator defined as follows

$$R\phi = \frac{1}{2}Tr[\sigma\sigma^T(x)]D_x^2\phi + \langle b(x), D_x\phi\rangle$$
$$S\phi = \int_E \big(\phi(t, x + c(x,e)) - \phi(t,x) - \langle\nabla\phi(t,x), c(x,e)\rangle\big)\lambda(de)$$

- $\mathcal{B}$ is an integral operator defined as

$$\mathcal{B}\phi = \int_E \big(\phi(t, x + c(x,e)) - \phi(t,x)\big)\gamma(x,e)\lambda(de).$$

- $\dfrac{\partial}{\partial n}$ defined by

$$\frac{\partial\phi}{\partial n} = \langle\nabla\phi, \nabla\Phi(x)\rangle, \quad \forall x \in \partial G.$$

From the viewpoint of non-linear Feynman-Kac's formula, proposed by [3], the above IPDE (1) should be related to the following decoupled forward-backward SDE with jumps :

$$\begin{cases} (i)\ X_s^{t,x} = x + \displaystyle\int_t^{t\vee s} b(r, X_r^{t,x})dr + \int_t^{t\vee s}\sigma(r, X_r^{t,x})dW_r + \int_t^{t\vee s}\int_U c(r, X_{r^-}^{t,x}, e)\tilde{N}(dr, de), \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + \displaystyle\int_t^{t\vee s}\nabla\Phi(X_r^{t,x})d\kappa_r^{t,x}, \\ (ii)\ \kappa_s^{t,x} = \displaystyle\int_t^{t\vee s}\mathbb{1}_{\{X_r^{t,x}\in\partial D\}}d\kappa_r^{t,x}. \end{cases} \tag{2}$$

and

$$\begin{cases} (i)\ Y_s^{t,x} = H(X_T^{t,x}) + \displaystyle\int_s^T f(r, X_r^{t,x}, Y_r^{t,x}, Z_r^{t,x}, V_r^{t,x})dr + \int_s^T g(r, X_r^{t,x}, Y_r^{t,x})d\kappa_r^{t,x} + K_T^{t,x} - K_s^{t,x} \\ \qquad\qquad\qquad\qquad\qquad - \displaystyle\int_s^T Z_r^{t,x}dW_r - \int_s^T\int_U V_r^{t,x}(e)\tilde{N}(dr, de),\ t \le s \le T, \\ (ii)\ Y_s^{t,x} \ge \ell(s, X_s^{t,x}),\ \text{ and } \displaystyle\int_t^T (Y_{s^-}^{t,x} - \ell(s, X_{s^-}^{t,x}))dK_s^{t,x} = 0, \quad \mathbb{P}-a.s. \end{cases} \tag{3}$$

where $W$ is a standard Brownian motion, $\tilde{N}$ is a compensated Poisson random measure and $\kappa$ is a continuous increasing progressively measurable process and all processes are defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ equipped with a filtration carrying $W$ and $N$.

The main purpose of this work is to generalize the results of [1] to study the IPDE (1) in a general setting. Our result will enjoy the feature that the nonlinear functions $f$ and $g$ in (3) will be supposed to satisfy stochastic conditions of monotonicity, Lipschitz and linear growth. The monotone (or Lipschitz) coefficient is indeed allowed to be an adapted process and not bounded. Then we cannot apply the standard results under standard monotone and Lipschitz conditions.

To accomplish the main goal, we must first demonstrate the existence and uniqueness of the solution of generalized BSDE (3) by improving the generator's conditions to satisfy stochastic monotonicity and stochastic Lipschitz assumptions on $f$ and $g$. This BSDE is solved in two steps. The first, when the generator $f$ does not depend on the variable $z$ and $v$, we use a Yosida approximation of monotone functions. The second step is resolved by the method of contraction mapping. We prove a comparison result of reflected generalized BSDE with jumps in this case. Our framework also proves the viscosity solution's continuity. The existence of the viscosity solution of (1) is shown by the same method in [2] with some modifications.

## 2 Reflected Generalized BSDE with jumps

The main assumption for this paper is to consider $f$ and $g$ in (3) such that : $\forall s \in [t,T]$, and $(y,z,v),(y',z',v') \in \mathbb{R} \times \mathbb{R}^d \times \mathcal{L}^2_\lambda$, there exist four $\mathcal{F}_t$-adapted processes $\alpha : \Omega \times [0,T] \to \mathbb{R}$, $\beta : \Omega \times [0,T] \to \mathbb{R}^{-*}$ and $\theta, \eta : \Omega \times [0,T] \to \mathbb{R}^{+*}$ such that:

$(i)$ $(y-y')\left(f(s,X_s^{t,x},y,z,v) - f(t,y',z,v)\right) \leq \alpha_s |y-y'|^2$,

$(ii)$ $(y-y')\left(g(s,X_s^{t,x},y) - g(s,X_s^{t,x},y')\right) \leq \beta_s |y-y'|^2$,

$(iii)$ $|f(s,X_s^{t,x},y,z,v) - f(s,X_s^{t,x},y,z',v')| \leq \theta_s |z-z'| + \eta_s \|v-v'\|_\lambda$.

Then

**Theorem 2.1.** *The generalized reflected BSDE* (3) *has a unique solution* $(Y_s^{t,x}, Z_s^{t,x}, V_s^{t,x}, K_s^{t,x})_{t \leq s \leq T}$.

## 3 Obstacle problem for IPDE with non-linear Neumann boundary condition

We prove that the deterministic function $u(t,x)$ defined by means of the representation of Feynman Kac's formula of the process $Y^{t,x}$, i.e.

$$Y_s^{t,x} = u(s, X_s^{t,x}) \ \forall s \in [t,T] \ \text{ and then } \ u(t,x) = Y_t^{t,x}.$$

is the unique continuously viscosity solution of (1).

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# Session 2 : Numerical Methods, Discrete Mathematics and Embedded Computing

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# Finite element method for elliptic problems involving the operators satisfying non-polynomial growth.

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**Abstract**
In this paper, we shall study the polynomial approximation in more general setting namely to consider the Orlicz-Sobolev spaces $W^k L_M(\Omega)$. A generalization of Cea's Theorem is established, also we consider the application of the error estimates and the convergence for the $M$-Laplacian, then finding the approximation solution to the Dirichlet problem associated to $M$-Laplacian by using the finite element method.

*Keywords :* Finite element method, Orlicz spaces, $M$-Laplacian, Interpolation operators.

## 1 Introduction

Let $\Omega$ be a bounded domain in $\mathbb{R}^N$, for $1 < p < +\infty$ and $f \in W^{-1,p'}(\Omega)$, with $\frac{1}{p} + \frac{1}{p'} = 1$, let us consider the following Dirichlet problem:

$$-\operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) = f \text{ dans } \Omega.$$

Since 1975 several numerical studies have been carried out, by using the finite element method of this problem. The first work in this direction we find Marrocco-Glowinski [**?**] in 1975 and Ciarlet [**?**] in 1979, then the work of Chow in 1989 about the scheme of finite element method which showed that we could improve error estimates using the fact that the exact solution of the problem and the approximate solution are minimum of some convex functional. Moreover Liu-Barrett [**?**] in 1993 established some improvements on the error estimates which the singularity of the operator occurs only near the points where the gradient of the solution vanishes.

In all the work cited earlier, the $p^{th}$ power sets obviously the $N$-function $M(t) = \frac{|t|^p}{p}$ and the problem can be written as:

$$-\operatorname{div}\left(\frac{m(|\nabla u|)}{|\nabla u|}\nabla u\right) = f \text{ in } \Omega .$$

When $N$-function $M$ is not necessarily polynomial function this problem can not be formulated in the classical Sobolev space, but rather in the Orlicz-Sobolev space.

In this paper, we propose an approximation study of this problem by finite element method, we extend the fundamental theorems of finite element in Orlicz spaces i.e the theorem of Cea, we show the convergence of this scheme and thus we generalize the results of Glowinski and Marrocco [**?**] obtained for the Laplace operator by finite element in usual Sobolev space in a more general functional setting (the Orlicz-Sobolev spaces).

The approximation of function in Sobolev spaces by a function in finite element spaces has been well studied in the setting of standard Sobolev spaces $W^{k,p}(\Omega)$ with $1 \leq p \leq +\infty$, $k \in \mathbb{N}$, see e.g [**?**], [**?**], [**?**] and [**?**].

Nevertheless, little work is known for error estimates in the context of Orlicz-Sobolev spaces $W^kL_M(\Omega)$, for this reason, we will show the classical estimates for the interpolation error in $W^kL_M(\Omega)$, where we study the local and global interpolation estimate, then, we show the local and global interpolation estimate.

Finally this work is organized as follows: In section 2 we recall some well-knowns preliminaries and results of Orlicz-Sobolev Spaces. Section 3, we shall proof some properties of $M$-Laplacian. Section 4 we will show the classical estimates for the interpolation error in more general setting namely to consider the Orlicz-Sobolev spaces $W^kL_M(\Omega)$, then we show the local and global interpolation estimate. Section 5, we will study the finite element error estimate for $M$-Laplacian operator where we establish a generalization of Cea's Theorem and we prove the modular convergence of the gradient, then we present the existence result and its proof.

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# L(2,1)-labeling number and upper traceable number of circulant graphs

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**Abstract**

An $L(2,1)$-*labeling* of a graph $G$ is an assignment of nonnegative integers to the vertices of $G$ such that adjacent vertices get numbers at least two apart, and vertices at distance two get distinct numbers. The $L(2,1)$-*labeling number* of $G$, $\lambda(G)$, is the minimum range of labels over all such labelings.

For a graph $G$ of order $n$ and for a linear ordering $s : (x_1, x_2, \dots, x_n)$ of its vertices, let $d(s) = \sum_{i=1}^{n-1} d_G(x_i, x_{i+1})$, where $d_G(x_i, x_{i+1})$ denotes the distance between the vertices $x_i$ and $x_{i+1}$ in $G$. The *upper traceable number* of $G$, denoted $t^+(G)$, is $t^+(G) = \max d_G(s)$, where the maximum is taken over all linear orderings $s$ of vertices of $G$.

In this paper, we provide exact values for the $L(2,1)$-labeling number as well as the upper traceable number of circulant graphs $C_n(S)$, i.e., graphs with the set $\{0, 1, \dots, n-1\}$ of integers as vertex set and in which two distinct vertices $i, j \in \{0, 1, \dots, n-1\}$ are adjacent if and only if $|i-j|_n \in S$, where $|x|_n = \min(|x|, n-|x|)$ is the cyclic absolute value of an $x \in \mathbb{Z}$.

*Keywords :* $L(2,1)$-labeling number, upper traceable number, circulant graph.

## 1 Introduction

The $L(c_1, c_2, \dots, c_t)$-labeling problem has been considered as a general model for the frequency assignment problem in radio networks, where the goal is to assign radio frequencies to each transmitter in a way that the interference is prohibited between transmitters that are geographically close-the closer the transmitters are, the stronger the interference is.

Due to its difficulty, many particular cases of this general problem have been studied. Among all, labelings with constraints at two distances, i.e., $L(h,k)$-labelings and particularly $L(2,1)$-labeling introduced by Griggs and Yeh [1] have been the subject of many works. Formally, an $L(2,1)$-labeling of a graph $G$ is an assignment $f : V(G) \to \mathbb{Z}^+ \cup \{0\}$ such that

$$|f(x) - f(y)| \geq \begin{cases} 2, & \text{if } xy \in E(G), \\ 1, & \text{if } d_G(x,y) = 2, \end{cases}$$

where $d_G(x,y)$ denotes the distance between two vertices $x$ and $y$ in $G$. The $L(2,1)$-*labeling number* of $G$, $\lambda(G)$, is the minimum range of labels over all such labelings.

As shown in [2], the $L(2,1)$-labeling number is related to two other graph parameters: the *upper Hamiltonian number* [3] of a graph $G$ of order $n$, denoted by $h^+(G)$, is the maximum of $\sum_{i=0}^{n-1} d_G(x_i,x_{i+1})$, where $(x_1,x_2,\ldots,x_n)$ is a linear ordering of its vertices, and $d_G(x_i,x_{i+1})$ denotes the distance between the vertices $x_i$ and $x_{i+1}$ in $G$. The *upper traceable number* [4], denoted by $t^+(G)$, is obtained from $h^+(G)$ by ignoring the distance between the first and the last vertex: $t^+(G) = \max \sum_{i=0}^{n-2} d_G(x_i,x_{i+1})$.

Král et al. in [3] showed that the problem of determining the upper Hamiltonian number of a graph is *NP*-hard. The same method can be used to prove that computing the upper traceable number is also an *NP*-hard problem. As a result, we investigate this graph parameter for graphs with regular structure.

In this work we focus on circulant graphs. The choice of this particular class of graphs was made because of their regular structure. They form a very interesting family of graphs that can be described by only two integer parameters. Furthermore, their regular structure led them to be commonly used in interconnection networks which have applications in many domains such as the computer network design, telecommunication networking, and distributed computation.

**Definition 1.1.** *For $n \in \mathbb{N}$ with $n \geq 4$, let $S = \{s_1,s_2,\ldots,s_t\}$ where $s_i$ $(i = 1,2,\ldots,t)$ are positive integers such that $1 \leq s_1 < s_2 < \ldots < s_t \leq \lfloor \frac{n}{2} \rfloor$. The circulant graph $C_n(S) = (V,E)$ has the set $V = \{0,1,\ldots,n-1\}$ of integers as a vertex set and in which two distinct vertices $i, j \in \{0,1,\ldots,n-1\}$ are adjacent if and only if $|i-j|_n \in S$, where $|x|_n = \min(|x|, n-|x|)$.*

## 2 Main Results

The following result provides necessary and sufficient conditions for any undirected graph of order $n$ to have exact values for the upper traceable number.

**Lemma 2.1.** *Let $G$ be a graph of order $n$ and diameter 2.*

$$t^+(G) = 2(n-1) \text{ if and only if } \overline{G} \text{ contains an Hamiltonian path.}$$

Next we discuss upper traceable numbers and $L(2,1)$-labeling numbers of circulant graphs with connection sets $S = \{1,2,\ldots,\lfloor \frac{n}{2} \rfloor\}$, $S \backslash \{\lfloor \frac{n}{2} \rfloor\}$, $S \backslash \{a\}$ and $S \backslash \{a,b\}$.

### 2.1 The graph $C_n(S \backslash \{\lfloor \frac{n}{2} \rfloor\})$

**Lemma 2.2.** *For each integer $n \geq 6$, $diam(C_n(S \backslash \{\lfloor \frac{n}{2} \rfloor\})) = 2$.*

**Theorem 2.3.** *For each integer $n \geq 6$,*

$$t^+(C_n(S \backslash \{\lfloor \frac{n}{2} \rfloor\})) = \begin{cases} 2(n-1), & \text{if } n \text{ is odd,} \\ \frac{3n}{2} - 1, & \text{if } n \text{ is even.} \end{cases}$$

**Corollary 2.4.** *For each integer $n \geq 6$,*

$$\lambda(C_n(S \backslash \{\lfloor \frac{n}{2} \rfloor\})) = \begin{cases} n-1, & \text{if } n \text{ is odd,} \\ \frac{3n}{2} - 2, & \text{if } n \text{ is even.} \end{cases}$$

### 2.2 The graph $C_n(S\backslash\{a\})$

The integer $d$ denotes $gcd(n,a)$ where $a \in S$.

**Lemma 2.5.** *For each integer* $n \geq 6$, $diam(C_n(S\backslash\{a\})) = 2$.

**Theorem 2.6.** *For each integer* $n \geq 6$*,*

$$t^+(C_n(S\backslash\{a\})) = \begin{cases} 2(n-1), & \textit{if } d = 1, \\ 2n-d-1, & \textit{if } d \neq 1. \end{cases}$$

**Corollary 2.7.** *For each integer* $n \geq 6$*,*

$$\lambda(C_n(S\backslash\{a\})) = \begin{cases} n-1, & \textit{if } d = 1, \\ n+d-2, & \textit{if } d \neq 1. \end{cases}$$

### 2.3 The graph $C_n(S\backslash\{a,b\})$

The integer $d$ denotes $gcd(n,a,b)$ where $a,b \in S$.

**Lemma 2.8.** *For each integer* $n \geq 10$, $diam(C_n(S\backslash\{a,b\})) = 2$.

**Theorem 2.9.** *For each integer* $n \geq 10$*,*

$$t^+(C_n(S\backslash\{a,b\})) = \begin{cases} 2(n-1), & \textit{if } d = 1, \\ 2n-d-1, & \textit{if } d \neq 1. \end{cases}$$

**Corollary 2.10.** *For each integer* $n \geq 10$*,*

$$\lambda(C_n(S\backslash\{a,b\})) = \begin{cases} n-1, & \textit{if } d = 1, \\ n+d-2, & \textit{if } d \neq 1. \end{cases}$$

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# Review of articles on Automatic Arabic diacritization

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**Abstract**

Arabic texts absence of diacritics is a major problem that encounter Arabic natural language processing. In fact, the majority of Arabic texts are written without diacritics, which make reading difficult for computer programs. Many researches were conducted in order to find the optimal way to automatically diacritize Arabic texts. Various techniques were employed for this purpose. Three main categories are named: Linguistic, machine learning and hybrid. This paper presents a literature review of most indexed and cited studies with different methods for Arabic diacritization, published in the last quarter of century.

*Keywords :* NLP, Diacritization, Machine Learning

## 1 Introduction

Over the last quarter century, Arabic Natural Language Processing (NLP) has interested many researchers around the world. Lack of diacritics in texts is one of the most common problems in Arabic NLP. Diacritics are essential for reading Arabic texts. Indeed, they indicate different types of short vowels, nunation and the absence of vowels. This issue makes it difficult for computer programs and non-native speakers to read Arabic text without diacritics, unlike native speakers. Numerous researches and studies were conducted in order to find the best approach to automatically diacritize Arabic texts. In this paper, we present a literature review of most indexed and cited articles that addressed this topic with different methods. The structure of this paper is as follows: the methods utilized in the chosen articles are covered in the second section; a conclusion is provided in the third section.

## 2 Different methods for Arabic diacritic restoration

Numerous methods were used in order to find the best way to restore Arabic diacritics. These methods can be divided into three categories: linguistic, statistic and hybrid methods. Each category has its pros and cons.

## 2.1 Linguistic methods

Since Arabic diacritization is closely relative to grammar and syntactic rules. Some researchers have used linguistic methods to diacritize Arabic texts. These types of approaches require a wild linguistic knowledge in Arabic to use Arabic features for the right diacritization of the world.
Among these studies is the research curried out by Debili et al. in [5]. This work studied the relationship between grammatical tagging and automatic diacritization of undiacritized texts. Experiments on a text of 25,000 occurrences yielded 77% of the occurrences, while 23% remain ambiguous. Alserag is another linguistic diacritization system proposed in [4] that consists of three modules and is controlled by 13 linguistic rules. Word error rate (WER) is 18.63%, while Diacritic Error Rate (DER) is 8.68%. The system's diacritization process includes using a dictionary with partial diacritization, which could affect its performance. A recent linguistic diacritization system for Arabic, named Arabix-Unitex, was developed in [2]. It uses 24 rules to distinguish between partially diacritized [1] or fully diacritized [2] words, and offers a list of words that are mutually compatible and fully diacritized for each word. The constructed lexicon has 76000 fully diacritical lemmas, and inflection is used to create 6 million different shapes. The system covers more than 99% of the words used in major newspapers.

## 2.2 Statistic methods

In NLP tasks, machine learning approaches have become more and more popular over time. With machine learning approaches it is possible to resolve a linguistic problem, such as Arabic texts diacritization, without being specialized in linguistics. Many papers have proposed machine learning approaches for the automatic Arabic diacritization subject.
Some studies have given concrete and precise results by calculating the evaluation measures. As in the case of the paper [13]. In this paper, a maximum entropy approach was proposed to restore diacritics in an Arabic text. It incorporates lexical segment-based and part-of-speech tag features, and the achieved diacritic, segment and word error rates are 5.1%, 8.5%, and 17.3%. Under the case-ending-less mode, they are 2.2%, 4.0% and 7.2%. In [6], a recurrent neural network (RNN) based approach for automatic Arabic diacritization is presented. The model is trained to transcribe undiacritized Arabic text with fully diacritized sentences. When preprocessed with error correction techniques, the network achieves peak performance, reducing diacritic error rate by 25%, word error rate by 20%, and last letter diacritic error rate by 33%.
Another research described the results without giving the evaluation with metrics, as in [9]. Indeed, this work compared three Deep Learning models to address the issue of Arabic language automatic diacritical. The model with an architecture-based encoders and decoders gave the best performance in terms of diacritic and word error rate. The models can be enhanced by experimenting with other hyperparameters. the researchers in [8] have addressed a subproblem of the Arabic automatic diacritization, which is the non-questionable knowledge generated acquired from the training phase by machine learning models. For this purpose, regularized decoding and Adversarial training are proposed. Experiments on two corpora (ATB and Tashkeela) reveal that even with self-generated information, this model can learn diacritics and exceeds all previous studies on the subject.

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[1] Any diacritization scheme (DS), where a subset of letter is diacritized. There are 4 types of DSs: an inflectional DS marking the verb passivization only, another inflectional DS encoding both case and mood, a lexical DS marking the words with Shaddah and another lexical DS marking only the words with Sukun

[2] All the letters are diacritized

## 2.3 Hybrid methods

Both linguistic and machine learning approaches for automatic Arabic diacritization have their own pros and cons. For this reason, many researchers have proposed hybrid approaches that combine linguistic and machine learning methods, to resolve Arabic diacritic restoration. These approaches can benefit from the advantages of both methods.

the researchers in [11] present a diacritization system using a cascade of probabilistic finitestate transducers trained on the Arabic treebank and combining a word-based language model, a letter-based language model, and a basic morphological model. When ignoring the end-case, the best WER and DER achieved on the Al-Hayat corpus are respectively 7.33% 6.35%. The best WER and DER achieved considering the end-case, on the Al-Hayat corpus, are respectively 23.61% 12.79%.

One of the studies that used SVM for statistical Arabic diacritization is the work in [10]. Indeed, this work extended the diacritization system MADA by using a tagger and lexeme language model. SVM-Tool was used as the machine learning tool, without Viterbi decoding. The classifiers were trained on the exact training set defined in [13], and the error was decreased by 17.2% for WER, but 10.9% for DER. Another study that used SVM is in [1]. Their approach is based on the retrieval of the lexicon, the bigram and SVM statistical priority techniques. Case endings are treated as a post-processing task using syntactic information. According to the article, the overall performance of this diacritization system is comparable to the best diacritization model reported at the time.

Many hybrid Arabic diacritization systems have used MADAMIRA tool for morphological analysis and disambiguation of Arabic. the study in [3] is one of them. In this work the researchers used a classifier decision tree, the MADAMIRA analyzer, and linguistic rules. The algorithm was built using the Penn Arabic Treebank (PATB) dataset and improved word diacritization accuracy by 2.5% absolute on all words and 5.2% absolute on nominal values. This study will be followed up on to construct integrated models of morphological disambiguation and syntactic analysis. Syntactic tree errors account for 31% of errors. the researchers in [12] also proposed a hybrid approach based on a Recurrent Neural Network assisted by MADAMIRA. This technique was evaluated using the LDC ATB3 dataset and had an 8.40% word error rate and 2.39% diacritic error rate.

In [7], the researchers used a pipeline-based technique that included three components: a deep learning model, a character-level rule-based corrector, and a statistical corrector on the word level. The Tashkeela dataset was utilized for training and testing the model, and the results showed that when all Arabic letters were considered, the DER was 3.39% and the WER was 9.94%. When the last letter of each word was ignored, the DER was 2.61% and the WER was 5.83%.

# 3 Conclusion

Various methods were used to automatically diacritize Arabic texts. Linguistic, machine learning and hybrid are the three main categories of these methods. Linguistic techniques preserve the Arabic language's peculiarities, while machine learning algorithms are affected by the quality and size of the datasets. Hybrid approaches that combine many strategies produce the best outcomes. The majority of research used full diacritization. Morphological and syntactic diacritization must be considered to ensure the accomplishment of a full diacritization, but syntactic diacritization suffers from low accuracy. Despite the efforts put forward in this area, more may always be done.

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# Packing chromatic number of iterated Mycielskians

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**Abstract**

For a graph $G$, the packing chromatic number of $G$, denoted by $\chi_\rho(G)$, is the smallest integer $k$ such that there exists a coloring $f : V(G) \mapsto \{1,\dots,k\}$ of $G$ where every two distinct vertices $u$ and $v$ such that $f(u) = f(v)$ are at pairwise distance at least $f(u)+1$. This number is known to be quite large, even unbounded for simple classes of graphs. In this paper, we study graphs $G$ for which $\chi_\rho(\mu^t(G)) = 2^t\chi_\rho(G)$ for all $t \geq 1$, where $\mu^t(G)$ stands for the $t$-iterated Mycielskian of $G$. We show that a first natural upper bound of $\chi_\rho(\mu^t(G))$ is $2^t(|V(G)| - \alpha(G)+1)$ for any graph $G$ where $\alpha(G)$ is the independence number of $G$. This bound is rounded to $2^t\chi_\rho(G)$ if the diameter of $G$ is two. If moreover the graph $G$ belongs to $\mathcal{P}$, class of graphs which vertex set can be partitioned into $X \cup Y$ such that $Y$ is an independent set, $|X| < |Y|$ and there is an $|X|$-matching $M$ such that $M = \{xy : x \in X, y \in Y\}$, then $\chi_\rho(\mu^t(G)) = 2^t\chi_\rho(G)$. Two examples of such graphs are given. Also, we propose a special transformation of a family $(T_n)_{n\geq 5}$. Moreover, $T_n$ is a maximal triangle-free graph with typical structure as described by Balogh et al. [2].
*Keywords :* Packing coloring, Packing chromatic number, Maximal triangle-free graphs, Mycielskians, Iterated Mycielskians.

## 1 Introduction and preliminaries

Through this paper, the vertex set, edge set and the order of a graph $G$ will be denoted by $V(G)$, $E(G)$ and $|G|$, respectively. The number of vertices of a subset $A$ of $V(G)$ will be also called the *order* of $A$ and denoted by $|A|$ and we set $G[A]$ for the subgraph induced by vertices of $A$. For $x \in V(G)$, the set $N_G(x)$ of all adjacent vertices of $x$ is called the *open neighborhood* of $x$. We use the usual notations $\omega(G)$, $\alpha(G)$ and $\chi(G)$ to denote the clique number, the independence number and chromatic number of a graph $G$. The complete graph with order $n$ is denoted by $K_n$. The complete bipartite graph with partitions orders $m$ and $n$ is designated by $K_{m,n}$. The notation $\overline{G}$ is used for the complement graph of a graph $G$. A *matching* of $G$ is a subset $M$ of $E(G)$ such that no two edges of $M$ have a vertex in common (i.e an independent set of edges). A matching $M$ covering $n$ vertex will be said *n-matching*. A matching $M$ is said a *perfect matching* if $M$ covers all vertices of $G$, i.e $M$ is a $|G|$-matching. The *join* of two graphs $G_1$ and $G_2$, denoted by $G_1 + G_2$, is the graph obtained from the disjoint copies of $G_1$ and $G_2$ by connecting every vertex of $G_1$ to every vertex of $G_2$. A graph is said to be *triangle-free* (TF) if no two adjacent vertices are adjacent to a common vertex.

A. Zykov [17] and J. Mycielski [14] were two of early mathematicians to provide iterative constructions of families of large triangle-free graphs. For a finite set of graphs $G_1 = (V_1, E_1), G_2 = (V_2, E_2), \ldots, G_n = (V_n, E_n)$, the *Zykov product of graphs* $G_1, G_2, \ldots, G_n$, denoted by $\mathcal{Z}(G_1, G_2, \ldots, G_n)$, is the graph $Z = (V, E)$ with vertex set partition $V = X \cup Y$ such that $X := \bigcup_{i=1}^{n} V_i$ and $Y$ is the set of all $n$-tuples from the cartesian product of subsets $V_1, \ldots, V_n$, i.e $Y := \prod_{i=1}^{n} V_i$. We denote by $y_j$ each $n$-tuple of $Y$, $j \in \{1, \ldots, |Y|\}$ and let $y_j(k)$ be the $k^{th}$ component of $y_j$, $k \in \{1, \ldots, n\}$. If $x_k$ is a component of $y_j$ we write $x_k \in y_j$. The edge set $E$ is defined by

$$E = \bigcup_{i=1}^{|X|} \left( \bigcup_{j=1}^{|Y|} \{x_i y_j : x_i \in y_j\} \right).$$

That is, $E$ is constructed, in addition to $\bigcup_{i=1}^{n} E_i$, by joining each vertex $y_j$ from the subset $Y$ to precisely vertices $y_j(k)$ from $V_k$, $1 \leq k \leq n$.

Note that the construction of $\mathcal{Z}(G_1, G_2, \ldots, G_n)$ is the same even if the graphs $G_i$ are numbered differently. If all graphs $G_i$ are isomorphic to a same graph $G$, the graph $\mathcal{Z}(G_1, G_2, \ldots, G_n)$ will be denoted $\mathcal{Z}(G^n)$.

A triangle-free graph is said *maximal triangle-free* (MTF) if no edge may be added without producing a triangle. Every bipartite graph is triangle-free, but almost every triangle-free graph is bipartite [10] with some restrictions on graph size [3,15]. This leads to say that the most of triangle-free graphs are bipartite and subgraphs of a complete bipartite graph, but most of them are not maximal. Brandt et al. [5] presented an efficient algorithm for generating maximal triangle-free graphs. A table in their work shows that the number of MTF graphs grows exponentially in the number of vertices $n$ with $3 \leq n \leq 21$. For instance, the number of MTF graphs on 21 vertex is 2 911 304 940. More results are available on *The House of Graphs* [9] (available at `https://houseofgraphs.org/`).

The problem of *determining or estimating the number of maximal triangle-free graphs on n vertices* (as stated in [16]) suggested by Erdös was well studied in many papers [2,4]. Balogh and Petříčková [4] resolved this counting problem by proving a matching upper bound, that the number of maximal triangle-free graphs on vertex set $[n] := \{1, \ldots, n\}$ is at most $2^{n^2/8 + o(n^2)}$. Balogh et al. [2] asked the question on what is their typical structure and answered by showing the following theorem.

**Theorem 1.1** ( [2]). *For almost every maximal triangle free graph $G$ on $[n]$, there is a vertex partition $X \cup Y$ such that $G[X]$ is a perfect matching and $Y$ is an independent set.*

A *k-packing coloring* of a graph $G$ with vertex set $V$, for some integer $k$, is a mapping $f : V \to \{1, 2, \ldots, k\}$ such that for any two distinct vertices $u$ and $v$ from $V$ : if $f(u) = f(v) = i$, then $d_G(u, v) > i$, where $d_G(u, v)$ is the distance between $u$ and $v$ in $G$. The *packing chromatic number* $\chi_\rho(G)$ of a graph $G$ is the smallest integer $k$ such that the graph $G$ has a $k$-packing coloring [13]. A *k-packing colorable* graph is a graph such that $\chi_\rho(G) \leq k$. The decision problem related to computing the packing chromatic number is NP-hard in general, even when restricted to trees [11]. Many are the papers investigating the boundedness of the packing chromatic number from above by a constant for several classes of graphs. This question for the class of subcubic graphs was answered in the negative [1] after being considered in [7, 8, 12]. Results before 2020 on packing coloring of graphs were summarized in the survey [6]. A trivial upper bound of the packing chromatic number was given in the seminal paper [13] as follows.

**Proposition 1.2** ( [13]). *For every graph $G$,*

$$\chi_\rho(G) \leq |G| - \alpha(G) + 1,$$

*with equality if* $diam(G) = 2$.

Recall the well-known property that for any graph $G$ we have $\chi(\mu(G)) = \chi(G)+1$. Hence for all $t \geq 1$, $\chi(\mu^t(G)) = \chi(G)+t$. That is the number $\chi(\mu^t(G))$ grows linearly in terms of $\chi(G)$. For the packing coloring, we study in this paper the exponential growth of $\chi_\rho(\mu^t(G))$ in terms of $\chi_\rho(G)$. Precisely, we investigate graphs for which

$$\chi_\rho(\mu^t(G)) = 2^t\chi_\rho(G). \tag{1}$$

## 2 Main results

**Proposition 2.1.** *For all graph $G$, $\alpha(\mu^t(G)) \geq 2^t\alpha(G)$.*

**Definition 2.2.** *$\mathcal{P}$ denote the set of graphs which vertex set can be partitioned into $X \cup Y$ where $Y$ is an independent set, $|X| < |Y|$ and there exists an $|X|$-matching $M$ such that $M = \{xy : x \in X, y \in Y\}$.*

**Theorem 2.3.** *For all $t \geq 1$, if $G \in \mathcal{P}$, then $\alpha(\mu^t(G)) = 2^t\alpha(G)$.*

**Theorem 2.4.** *For all connected graph $G$ and all $t \geq 1$,*

$$\chi_\rho\left(\mu^t(G)\right) \leq 2^t\left(|G| - \alpha(G) + 1\right).$$

**Theorem 2.5.** *If $G$ a graph of diameter two such that $G \in \mathcal{P}$, then for all $t \geq 1$,*

$$\chi_\rho(\mu^t(G)) = 2^t\chi_\rho(G).$$

**Theorem 2.6.** *For all $n \geq 5$, there exists a large MTF with typical structure (good MTF) $G_n$ such that $\chi_\rho(\mu^t(G_n)) = 2^t\chi_\rho(G_n)$.*

The number of 'bad' maximal triangle-free graphs, is exponentially smaller than the number of maximal triangle-free graphs.

**Theorem 2.7.** *There exists a large MTF graph $G$ with non typical structure (bad graph) such that $\chi_\rho(\mu^t(G)) = 2^t\chi_\rho(G)$.*

We summarize in Table 1 graphs satisfying (1) of this paper with their properties.

| | TF | diameter two | MTF | good MTF | in $\mathcal{P}$ |
|---|---|---|---|---|---|
| $K_{m,n}$, ($m < n$) | yes | yes | yes | no | yes |
| $G + \overline{K_{\lvert G\rvert+1}}$, ($G$ is arbitrary) | no | yes | no | no | yes |
| $T_n$, $n \geq 5$ | yes | yes | yes | yes | yes |

Table 1: A table of this paper's graphs satisfying (1) and theirs properties.

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# ANALYSIS RESULTS FOR DYNAMIC CONTACT PROBLEM THERMOPIEZOELECTRIC MATERIALS

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**Abstract**

We consider a mathematical model which describes the dynamic process of contact between a piezoelectric body and a rigid foundation. We model the material's behavior with a thermo-electro-viscoelastic constitutive law. The friction is modeled with Tresca's friction law. We derive variational formulation for the model which is in the form of a system involving the displacement field, the electric potential and the temperature. We prove the existence of a unique weak solution to the problem. The proof is based on regularization method followed by Faedo-Galerkin's method and fixed point theorem.

*Keywords :* Keywords:Thermopiezoelectric; Weak solvability; Regularization method; Faedo-Galerkin method; Banach fixed point theorem.

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# A brief overview of the applications of AI-powered Visual IoT systems in agriculture

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**Abstract**

The integration of Internet of Things (IoT) and Artificial Intelligence (AI) technologies has revolutionized the field of agriculture by enabling advanced monitoring systems for crop quality, disease detection, and yield estimation. In this paper, we provide a brief overview of recent research works that leverage AI-powered visual IoT systems in the agriculture field. We then discuss the challenges and opportunities encountered by the presented works. Lastly, we point out future directions towards the development of efficient and sustainable AI-powered IoT systems for agricultural monitoring.

*Keywords: Visual IoT, Artificial Intelligence, Agriculture, Internet of Things*

## 1 Introduction

Agriculture plays a vital role in ensuring food security and sustainable development. With the increasing global population and changing climatic conditions, the need for efficient and smart agricultural monitoring techniques has become paramount. In recent years, the integration of AI and IoT technologies has emerged as a promising solution for addressing the challenges in agriculture. Among the various AI-powered techniques, imagery-based AI has gained significant attention in the field of agriculture. The use of visual data from cameras and sensors, combined with AI algorithms, allows for real-time monitoring and analysis of agricultural processes. This enables farmers and agricultural practitioners to make data-driven decisions, optimize resource usage, and improve crop yields. Table 1 gives an overview of recent research themes in Visual IoT and a list of papers where the research corresponding to a certain theme is performed.

In this paper, we provide an overview of recent research papers that focus on the applications of AI-powered visual IoT systems in agriculture. We categorize these papers based on fields such as precision agriculture, crop health management, yield monitoring and management. We also provide challenges and opportunities that come with Visual IoT systems. This analysis aims to provide insights into the current state of the field and potential future research direction towards the development of efficient and sustainable AI-powered IoT systems for agricultural monitoring.

## 2 Applications of AI-Powered Visual IoT in Agriculture

Agriculture is a crucial sector that plays a significant role in providing food for the growing global population. With the advancement of technology, particularly in the fields of AI and IoT, there have been significant developments in the application of AI-powered visual IoT systems in agriculture. These systems leverage image recognition, video analysis, and data processing techniques to enable smart and automated monitoring and management of crops, yields, and resources in agriculture.

### 2.1. Precision agriculture

Yi-Wei et al. [1] discussed the use of IoT and AI to manage and control irrigation operations. The authors utilize image processing techniques and a convolutional neural network (CNN) to categorize soil moisture levels into high, normal, and low categories. They then apply a fuzzy control system to define the soil moisture interval based on the category of crop being planted. The results are sent to a control device for precise and comprehensive irrigation control. The use of AI and IoT in this manner has shown promising results in improving accuracy in irrigation and reducing water waste. Ganesh et al. [2] proposed a new agricultural robot, named agriBOT, that integrates smart sensors and AI logic to monitor crops and agricultural fields. The agriBOT acts as a drone to survey fields and is connected to the IoT for remote monitoring by farmers. It is able to collect data about weather, soil moisture and take photos of crops to detect leaf diseases. The authors introduce a machine learning strategy called Modified Convolutional Neural Scheme (MCNS) to analyze server data and predict climate conditions and crop details. The agriBOT can also identify plant leaf disease based on images captured. The proposed system is experimentally tested and proven to be efficient.

### 2.2. Crop Monitoring and Management

Murugamani et al. [3] worked on the development of wireless sensor systems that use AI and image processing techniques to detect and control crop diseases, as well as monitor soil quality, moisture, temperature, and chemical levels. These systems can provide real-time information to farmers through mobile applications, allowing for efficient and timely management of crop health. Another approach proposed by Ching-Ju et al. [4] involves AI and image recognition technology to develop real-time pest identification systems that predict the occurrence of pests and provide accurate location information to farmers. By reducing the amount of pesticides used, these systems help decrease environmental damage and improve overall agricultural economic value. Sindhu et al. [7] proposed an AI-powered wireless sensor system has been developed for detecting plant diseases in agriculture to reduce financial losses. The proposed system relies on CNN techniques to achieve accurate image classification. The study used a combination of a pretrained deep learning model (VGG19) and a contour feature-based method that employed the pyramid histogram of oriented gradient (PHOG). By merging the most effective features for classification, the fusion process resulted in an accuracy rate of up to 99.6%. This approach has practical applications for 5G technology, cloud computing, and the IoT. Alisha et al. [10] presented an AI-based wireless sensor system for agriculture that utilizes AI and IoT technologies to build advanced monitoring systems to ensure crop health and prevent damage. The paper addresses key steps such as image pre-processing, feature extraction, classification, and analysis of results. The integration of AI and IoT has led to significant advancements in the accuracy of agricultural tasks. The proposed system is applied on grape fields to monitor mainly two categories of plant diseases: Downy Mildew and Powdery Mildew. This was achieved using a Support Vector Machine (SVM) model, which yielded good results with an accuracy of 89%.

### 2.3. Yield Monitoring and management

The implementation of AI techniques has been proposed in several studies to monitor and manage crop yield. In the study presented by Yudhi et al. [5], the authors proposed the use of the grey level co-occurrence matrix (GLCM) method for feature extraction on digital images of cocoa beans. The GLCM method was found to be more reliable than the CNN method for feature extraction, and it was implemented on a low-performance computational device, demonstrating the potential for increasing the security of the food supply chain. The usage of the proposed GLCM textural features method has improved the feature extraction accuracy by 8.09% and 1.9% for SVM and XGBoost classifiers respectively. Savvidis et al. [9] proposes an edge computing approach that utilizes AI and IoT technologies to monitor apple orchard yield and detect apples for harvesting purposes. The approach uses a low-power information relay using Lorawan protocol and processes data on a battery-driven edge device on-site. The YOLOv4 framework is implemented on a single-board computer with a camera using custom-trained weights and achieved a performance of up to 66.89% for apple detection in complex dense environments. The preliminary results suggest the feasibility of this approach.

| AI techniques | Papers |
|---|---|
| SVM | [5], [10] |
| Yolo | [4], [9] |
| Xgboost | [5] |
| VGG19 | [7] |
| LSTM | [4] |
| RandomForest | [1] |
| CNN | [1] |

*Table 1: AI techniques used in the reviewed works*

## 3. Challenges and opportunities

The Internet of Things has evolved to encompass the Visual Internet of things, presenting both challenges and opportunities in terms of design and application. Design issues in Visual IoT stem from the nature of sensor data, particularly with video cameras, which consume more energy, require higher bandwidth, and produce non-trivial real-time data. Consequently, video cameras are often not battery powered and require electrical grid connections, limiting camera locations. In addition, transferring video streams from cameras also consumes more bandwidth than classical sensors, requiring new solutions optimized for operation in less resourceful communication networks. To address these challenges, Edge and Fog computing paradigms have emerged, allowing for processing or preprocessing data closer to the sensor and streaming only results to the cloud. Deep learning techniques have also been applied to Visual IoT to improve performance, though challenges remain in their integration. Nonetheless, the application of deep learning to Visual IoT holds great promise for future developments in the field.

## 4. Conclusion

This paper provides an overview of recent research on AI-powered visual IoT systems in agriculture. The integration of AI and IoT technologies allows for real-time monitoring and analysis of agricultural processes, enabling farmers to make data-driven decisions, optimize resource usage, and improve crop yields. We categorized recent research papers based on fields such as precision agriculture, crop health management, and yield monitoring and management. We finally discussed the challenges and opportunities that come with visual IoT systems and provide insights into potential future research directions.

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# The Application of Machine Learning in E-learning

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**Abstract**

E-learning is a constantly evolving field, allowing learners to access online courses from anywhere and at any time. However, with the increasing number of courses available online, it is becoming increasingly difficult for educators to provide a personalized experience for each learner. The use of Machine Learning techniques can help solve this problem by creating predictive models that can enhance the learner's experience.

In this article, we provide an overview of the state of the art of Machine Learning application in E-learning. We discuss different types of Machine Learning algorithms used in E-learning, such as classification, recommendation, and prediction. We also explore the advantages and limitations of using these techniques.
*Keywords :* machine learning, E-learning, predictive models,classification,recommendation.

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# MOOC's Learners classification : A behavioral generation framework based methodology

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**Abstract**

Since their beginning, Massive Open Online Courses (MOOC) have known great success and have managed to establish themselves with significant enrollment rates. However, this success was quickly disrupted by the drop-out phenomenon observed in the majority of MOOCs, which reaches 90% in some courses. Studying and understanding this phenomenon, and consequently determining the relevance of the efforts made to develop MOOCs, has led several researchers to propose predictive models of learners at risk of dropping out. On one hand, these models have been made relying on machine learning and the massive data generated by learners' navigation. On the other hand, these models only provide weekly predictions and do not give clear visibility about the overall course progress. We present in this paper a framework based on the recurrent neural networks' strengths which uses generator and predictor modules. Our framework allows not only the prediction of dropouts but also the generation of each learner's behaviors during the whole course since its first week. Besides, an OLAP analytical module proved great support for MOOC moderators to report on the learners' behavior at-risk to target their interventions and guide their support.

*Keywords :* MOOC, Instructor Support, Educational Data Analytics, Machine Learning, LSTM

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# A Comparative Study of Numerical Techniques for Solving Intuitionistic Fuzzy Differential Equations

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**Abstract**

This paper is devoted to the numerical comparison of methods applied to solve the intuitionistic fuzzy differential equations. Five numerical methods are compared with the exact solutions of the proposed problems. To demonstrate the validity of the comparisons, numerical examples are provided.

*Keywords :* Differential equations, Numerical methods, Intuitionistic fuzzy number.

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# Deep Learning approach for tomato leaf disease prediction

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**Abstract**

Through this work, we have made a comparative study of some deep learning techniques and particularly CNN models applied to public datasets in order to determine the adequate, fast and reliable method that allows the detection of diseases affecting the tomato leaves. Our choice is based on the three models YOLOv5, YOLOX and YOLOv7 which belong to the category of "One-stage detectors" known for their speed of inference and their important precision. We have reached a prediction accuracy score of 93,1% for tomato diseases. The final model is deployed as a mobile application for ease of use.

*Keywords :* Tomato plant disease; Precision agriculture; Deep learning; YOLO

## 1 Introduction

With a growing demand for food, agriculture is facing a major challenge to feed the entire planet. Plant diseases are one of the major issues leading to a significant reduction in the quality and quantity of plant production. Traditional methods for identifying these diseases are effective but require expert intervention and can be delayed, increasing the risk of crop loss. The use of deep learning would enable faster diagnosis of diseases and accelerate intervention. For high-quality agricultural production, precision agriculture is essential. Machine learning and deep neural networks have significantly improved the accuracy of object detection and recognition systems.

In the field of plant disease detection, several studies have used convolutional neural networks (CNNs) to achieve high levels of accuracy. Among these studies, Fujita et al. [1] proposed a classifier for cucumber diseases using CNNs and achieved an accuracy of 82.3%. Brahimi et al. [2] presented the CNN model as a learning algorithm for tomato disease classification, with an accuracy of 99.18%. Rangarajan et al. [3] used AlexNet and VGG16 to classify six different diseases and one healthy class of tomatoes, achieving classification accuracies of 97.49% and 97.23%, respectively, for 13,262 images. Finally, Qimei Wang et al. [4] developed methods for tomato disease detection based on deep convolutional neural networks and object detection models, achieving accurate and fast results for eleven types of tomato diseases.

This work aims to design and implement a machine learning system to automatically detect and classify tomato leaf diseases by analyzing images of diseased and healthy plants. The final model will predict the leaf's condition in real-time and provide a reliable and fast tool for fighting diseases, even for non-experts.

In this report we will introduce the methodology used to implement our target system, then we will present the obtained results and discusses their significance.

## 2 Methodology and results

### 2.1 Model description

There is a multiplicity of object detection algorithms that differ from each other depending on their accuracy, speed, hardware resources required, and even the number of classes supported. Indeed, in our case, we were interested in object detection algorithms using CNN detection models, they are divided into two large families, one that does object detection in two steps and the other in a single step. In our work, we opted for the single step model, especially the YOLO model family, because it uses the characteristics of the entire image to predict each bounding box. It also predicts all bounding boxes of all classes in an image simultaneously. This means that this network reasons globally about the whole image and all the objects it contains. YOLO design enables end-to-end learning and real-time speeds while maintaining high average accuracy. Since YOLO Models have many versions, we focused our work on the lattest one. Thus, we made the comparison between 3 versions, Yolov5, YoloX and Yolov7 models. There is a brief overview of the different models :

**Yolov5** : Is a single-stage object detector that uses anchor boxes to predict bounding boxes and class probabilities. It has a simple architecture that consists of a backbone network and a detection head. YOLOv5 has been shown to be faster than previous versions of YOLO while maintaining similar accuracy.

**YoloX** : is another single-stage object detector that was introduced in 2021. It uses a new anchor-free detection head that is designed to be more efficient than the anchor-based detection heads used in previous versions of YOLO. YOLOX also introduces a new training strategy called "self-training" that allows it to achieve state-of-the-art performance on several object detection benchmarks.

**Yolov7** : was introduced in 2022. It is based on previous versions of YOLO such as YOLOv4, Scaled YOLOv4, and YOLO-R. YOLOv7 introduces several new features such as a new backbone network, a new detection head, and a new training strategy called "progressive training" that allows it to achieve state-of-the-art performance on several object detection benchmarks

Deep learning is based on the use of huge amount of data. So, in our present case, we need a high number of images of sick and healthy tomato leaves to train our object detection models. Since collecting images in the field is a time-consuming task, we opted for the use of two public databases. **PlantVillage** and **PlantDoc**. We were satisfied with images of two classes of diseases (Tomato Late Blight, Tomato Septoria leaf spot ), in addition to a class that contains images of healthy tomato leaves. This choice is due to the fact that we must annotate images from PlantVillage (3000 images to annotate). In total, our database for object detection will consist of 3325 images. Each Image can have multiple bounding boxes depending on the leaves' health condition, at the end we have 4025 bounding boxes (annotations). The final dataset was divided into thee samples training, validation, and testing. Table 1 shows the total number of training, testing, and validation images.

| Training Sample | Validation Sample | Testing Sample | Total Sample |
| --- | --- | --- | --- |
| 2327 | 665 | item 333 | 3325 |

Table 1: Images number for train, validation and test

### 2.2 Results and discussions

After training the different models with our final dataset, we get the results illustrated in the Table 2 below. The Yolov5 model performs better than others with mAP0.5 score of 92.7%, followed by YOLOX-s 89.3%, YOLOv7-Tiny has a minimum value of 87.6%.

| | **mAP 0.5** | **mAP 0.5:0.95** | **Total Loss** |
|---|---|---|---|
| **YOLOv5s** | 92,7 % | 78,7% | 0.02 |
| **YOLOX-s** | 89,3 % | 76,8 % | 0,59 |
| **YOLOv7 Tiny** | 87,6 % | 72,1 % | 0,042 |

Table 2: Results obtained after training the 3 models

The YOLOv5 model has been chosen for the hyperparameter tuning step, the purpose of this operation is to find the best hyperparameter values to fine-tune the final model, thus increasing its accuracy. After running the hyperparameter tuning we managed to improve mAP0.5 scores by 0.4% to 93.1% and mAP0.5:0.95 by 3.6% to 81.55%.

After getting the final model weight, we decided to deploy our model in an Android application that will be able to detect the different diseases of tomatoes on the field. Thus, we need first to convert the model after training from PyTorch format **(.pt)** to Tensorflow Lite format **(.tflite)**. The generated file will be used in the Android application and can be deployed in a smartphone.

## 3 Conclusion

In this work, we aimed to develop an efficient and accurate method for early detection of tomato diseases using deep learning algorithms. We utilized images of tomato leaves from two datasets, PlantVillage and PlantDoc, and merged them to create a diverse and comprehensive dataset for training our models. We compared three versions of the popular YOLO (You Only Look Once) object detection model, namely YOLOv5, YOLOX, and YOLOv7. Among the three versions, YOLOv5 showed the highest accuracy, with a score of 92.7%. However, we didn't stop there and aimed to further improve the performance of our model. We used hyperparameter evolution techniques to fine-tune the model's hyperparameters and achieved a significant improvement, with the accuracy reaching 93.1%. Overall, our work demonstrates the effectiveness of deep learning algorithms, specifically the YOLO family of models, for tomato disease detection based on leaf images. We have achieved a high accuracy of 93.1% through hyperparameter evolution and developed a mobile app to make our system more practical and user-friendly. Our research contributes to the field of agricultural technology, providing a valuable tool for early disease detection in tomatoes, which can potentially improve crop yields and reduce losses due to diseases.

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# Enhancing Speech Emotion Recognition: A Focus on Energy Analysis in Six Frequency Bands with Attention Mechanism

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**Abstract**

The objective of Speech Emotion Recognition (SER) is to enable machines to comprehend human emotions based on audio input. However, the process of extracting and incorporating significant features from audio input remains a difficult challenge. As a solution, we suggest utilizing the attention mechanism. This approach recalculates the importance coefficients or weights of distinct features or components of the audio input by prioritizing certain parts over others. Consequently, this can boost the performance of the signal processing task.

*Keywords :* Attention mechanism, speech emotion recognition.

## 1 Introduction

Speech emotion recognition (SER) is a technology that focuses on developing a system capable of recognizing and analyzing emotions in speech signals through the use of various techniques, including feature extraction, signal processing, and machine learning. Its primary goal is to enable machines to understand and respond appropriately to human emotions, improving the quality of human-computer interaction.

Recently, emotion recognition from speech has become a critical subject, but it is a complex field that relies on various elements such as pre-processing techniques, feature extraction, and classification. Our work is inspired by several studies that use the attention mechanism to focus only on the most important parts, for example, in [4] the authors developed an end-to-end (e2e) learning framework with a multi-task learning (MTL) strategy and a self-attention layer to extract important representations for specific tasks. Their experiments on the Interactive Emotional Dyadic Motion Capture ( IEMOCAP) database showed that the proposed framework outperformed single-task-based e2e systems, but did not perform as well as baseline systems with classic hand-crafted features for arousal. Furthermore in [2] The authors observed an improvement in the performance of the speech emotion recognition (SER) system by using both the attention mechanism and Deep Canonical Correlation Analysis (DCCA) to jointly learn the parameters of the magnitude and phase features. These techniques helped to extract emotion-relevant features more effectively and led to a significant improvement in the unweighted accuracy (UA) metric on the IEMOCAP database. Also this study [3] improved a speech emotion

recognition (SER) model by combining Interspeech 2009 Emotion Challenge feature set (IS09) and mel spectrogram features with a long-term descriptor (LLD). The model utilized an attention mechanism, dense layers, and bidirectional LSTM. Evaluation on the IEMOCAP dataset showed a 3% improvement in both weighted accuracy (WA) and UA for the attention-LSTM-attention model (ALA) model.
In light of the findings, we have resolved to incorporate the attention mechanism into our existing approach [1], utilizing the accomplished steps in the process as a foundation for implementation.

## 2 Proposed work

Our study will focus on the Ryerson Audio-Visual Database of Emotional Speech and Song (RAVDESS), which consists of 7353 files. The database features 24 professional actors, half of whom are male and the other half female, who deliver two sentences in a neutral North American accent: "Kids are talking by the door" and "Dogs are sitting by the door". The speech and songs in the database comprise neutral, happy, sad, angry, fearful, surprised, and disgusted expressions, each produced at two levels of emotional intensity, namely normal and strong.
Our emotion recognition system requires three main components: the emotional database, the extraction of characteristic parameters that can reflect emotional information, and the application of these parameters as input to an attention mechanism with CNN (Convolutional Neural Network) for identifying the most relevant features. The weighted features are then computed, and the results are fed as input to an MLP (Multilayer Perceptron) classifier for recognizing the emotion (Figure 1).

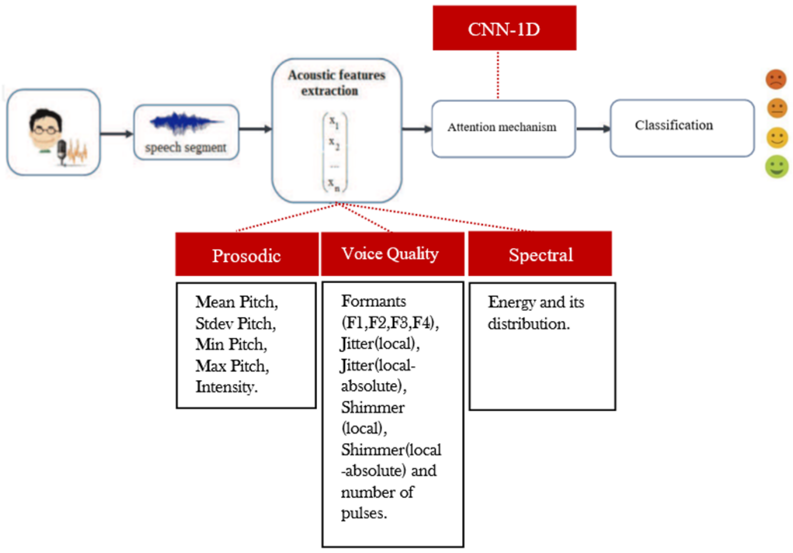


Figure 1: The Process of Recognizing Emotions.

## 3 Result and conclusion

We observed that after incorporating the attention mechanism, the results appear to be similar to our previous work [1], except for a slight improvement in negative emotions Figure 2, Figure 3. As future work, we aim to enhance the proposed architecture and compare the results with other databases.

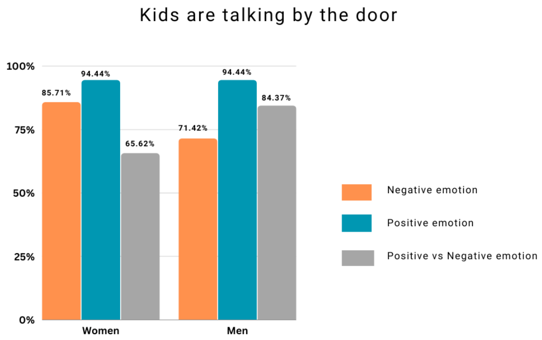


Figure 2: Analyzing the sentence "Kids are talking by the door ".

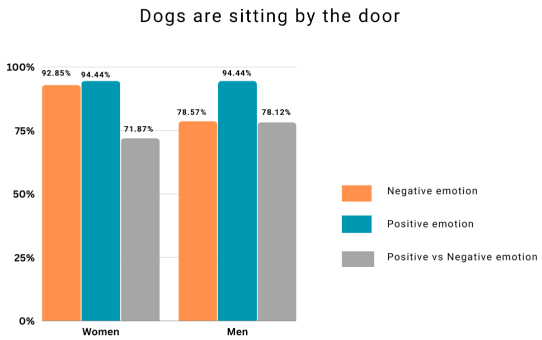


Figure 3: Analyzing the sentence "Dogs are sitting by the door".

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# A hybrid approach for solving differentiable unconstrained optimization problems

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**Abstract**

In the present work, the best characteristics of Particle Swarm Optimization (PSO) have been combined with the good local search characteristics of the Nonmonotone Spectral Gradient (NSG); the proposed algorithm is devoted to solving differentiable unconstrained optimization problems. The numerical results of solving benchmark problems assess the performance of our proposed algorithm.

*Keywords :* Particle Swarm Optimization; Nonmonotone spectral gradient method; Differentiable optimization; Metaheuristics

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## 1 Introduction

Optimization algorithms are classified into classical or deterministic and stochastic methods. Stochastic optimization refers to the minimization (or maximization) of a function in the presence of randomness in the optimization process. Common methods of stochastic optimization include stochastic approximation, stochastic programming, and metaheuristic methods.

Two major components of any metaheuristic algorithms are exploration and exploitation, or diversification and intensification. The goal is to diversify the search all over the search space and intensify the search in some promising areas. In other words: Intensification guides the method to deeply explore a promising part of the search space. In contrast, diversification aims at extending the search to different parts of the search space.

Inspired by the flocking and schooling patterns of birds and fish, Particle Swarm Optimization (PSO) was invented by Russell Eberhart and James Kennedy in 1995 [2].

In our proposed hybrid approach which we call HyPSOG [4], in every iteration of PSO, and under specific conditions, we perform an exploitation step by a variant of the gradient method.

In the following section, our approach is outlined, in section 2, numerical results are presented and the paper is concluded in section 3.

## 2 HyPSOG Method

**Particle Swarm Optimization** The basic PSO algorithm consists of three steps, namely, generating the particle's positions and velocities, velocity update, and finally, position update. A particle changes its position from one move (iteration) to another based on velocity updates. First, the positions, $x_i$, and velocities,$v_i$, of the initial swarm of particles are randomly generated using upper and lower bounds on the design variables values, in the second step, velocities and positions of all particles are updated, to persuade them to achieve better objective or fitness values, which are functions of the particles current positions in the design space.

The fitness function value of a particle determines which particle has the best global value in the current swarm, $p_g$, and also determines the best position of each particle over time, $p_i$, i.e. in current and all previous moves. The velocity update formula is given by:

$$v_{ij}^{k+1} = \omega v_{ij}^{k} + c_1 r_1 (p_{ij}^{k} - x_{ij}^{k}) + c_2 r_2 (p_{gj}^{k} - x_{ij}^{k}) \tag{1}$$

Here, $c_1$ and $c_2$ are the acceleration coefficients, and $r_1$ and $r_2$ are two uniformly distributed random numbers independently generated within [0, 1]. The inertia factor $\omega$ is used to balance the exploration and exploitation of the population. In the final step, we update the position:

$$x_i^{k+1} = x_i^k + v_i^{k+1} \tag{2}$$

**Nonmonotone Spectral Gradient method** The unconstrained minimization problem $\min_{x\in\mathbb{R}^n} f(x)$, where $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is a continuously differentiable function that has different iterative solving methods: If $x_k$ denotes the current iterate, and if it is not a good estimator of the solution $x_*$, a better one, $x_{k+1} = x_k - \alpha_k g_k$ is required. Here $g_k$ is the gradient vector of $f$ at $x_k$ and the scalar $\alpha_k$, is the step length. A variant of the steepest descent was proposed in [1], which is referred to as the 'Barzilai and Borwein' (BB) algorithm, where the step length $\alpha_k$ along the steepest descent $-g_k$ is chosen as in the Raliegh quotient

$$\alpha_k = \frac{s_{k-1}^T s_{k-1}}{s_{k-1}^T y_{k-1}}$$

where $s_{k-1} = x_k - x_{k-1}$ and $y_{k-1} = g_k - g_{k-1}$. This choice of step length requires little computational work and greatly speeds up the convergence of gradient methods.

Raydan in [5] has proved a global convergence of (BB) algorithm under a nonmonotone line search. In nonmonotone spectral gradient method, the iterate $x_k$ satisfies a nonmonotone Armijo line search (using sufficient decrease parameter $\gamma$ over the last $M$ steps),

$$f(x_{k+1}) \leq \max_{0\leq j\leq \min\{k,M\}} f(x_{k-j}) + \gamma\langle g_k, x_{k+1} - x_k\rangle \tag{3}$$

Here the function values are allowed to increase at some iterations. This type of condition (3) was introduced by Grippo, Lampariello, and Lucidi [3] and successfully applied to Newton's method for a set of test functions.

**Algorithm NSG** [5]

The algorithm starts with $x_0 \in \mathbb{R}^n$ and use an integer $M \geq 0$; a small parameter $\alpha_{min} > 0$; a large parameter $\alpha_{max} > 0$; a sufficient decrease parameter $\gamma \in (0,1)$ and safeguarding parameters $0 < \sigma_1 < \sigma_2 < 1$. Initially, $\alpha_0 \in [\alpha_{min}, \alpha_{max}]$ is arbitrary.

**Step 1.** Detect whether the current point is stationary

If $\|g(x_k)\| = 0$, stop, declaring that $x_k$ is stationary.

**Step 2.** Backtracking

**Step 2.1** Compute $d_k = -\alpha_k g_k$. Set $\lambda \longleftarrow 1$.

**Step 2.2** Set $\tilde{x} = x_k + \lambda d_k$.

**Step 2.2** If

$$f(\tilde{x}) \leq \max_{0\leq j\leq \min\{k,M\}} f(x_{k-j}) + \gamma\lambda\langle d_k, g_k\rangle \tag{4}$$

then define $\lambda_k = \lambda$, $x_{k+1} = \tilde{x}$, $s_k = x_{k+1} - x_k$, $y_k = g_{k+1} - g_k$ and go to Step 3.
If (4) does not hold, define $\lambda_{new} \in [\sigma_1, \sigma_2\lambda]$. Set $\lambda \longleftarrow \lambda_{new}$ and go to Step 2.2.
**Step 3**. Compute $b_k = \langle s_k, y_k \rangle$.
If $b_k \leq 0$, set $\alpha_{k+1} = \alpha_{max}$, else, compute $a_k = \langle s_k, s_k \rangle$ and

$$\alpha_{k+1} = \min\{\alpha_{max}, \max\{\alpha_{min}, a_k/b_k\}\}$$

**The proposed approch** In order to amplify the intensification aspect of PSO method, we propose to perform a local search, by NSG method, around $p_g$, in every iteration of PSO.

## 3 Numerical results and conclusion

In the following table we report the Standard Deviation by the algorithms for 10 runs. Here CPSOG

Table 1: Comparison between the Standard Deviations [4]

| F(n)[a] | PSO | CPSOG | HyPSOG |
|---|---|---|---|
| $f_1(40)$ | $3.476e+09$ | 3924.3487 | 0.0000002 |
| $f_2(2)$ | 33.274943 | 14.788509 | $5.217e-14$ |
| $f_3(2)$ | 88.243807 | 54.743889 | 0.0000088 |

[a] $n$ is the problem dimension, $f_1$, $f_2$ and $f_3$ are respectively the Extended Powell singular quartic function, Goldstein-Price's function and Shubert function.

algorithm is a classical hybridization between PSO and NSG in which the PSO carries out first a certain number of iterations, and then the NSG method, is applied to refine the approximations.

The numerical results of Table 1 show that, in general, the Standard Deviations given by HyPSOG are significantly smaller than those given by CPSOG and PSO. We conclude that the proposed method seems to be an interesting candidate for solving unconstrained differentiable optimization.

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# Variational analysis of a static thermo-electro-elastic contact problem with thermal Signorini's conditions

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**Abstract**

This paper explores a mathematical model that characterizes a static frictional contact between a thermo-piezoelectric body and an obstacle, the so-called foundation. The constitutive law of thermo-electro-elastic behavior is assumed to be nonlinear and features the nonlinear elastic constitutive law of Hencky. To describe the contact, a temperature-dependent Signorini conditions and a variant of Coulomb's friction law with a slip-dependent friction coefficient are used. A coupled system is formulated for displacement field, electric potential and temperature to address the problem. A variational formulation is established for the model and the existence of a unique weak solution to the problem is demonstrated. The proof relies on a abstract result concerning the existence and uniqueness of solutions for elliptic quasi-variational inequalities, as well as the use of Banach fixed point arguments.

*Keywords:* thermo-piezoelectric body, Hencky's law, Signorini contact conditions, Coulomb friction law, variational formulation, weak solution, elliptic quasi-variational inequalities, Banach fixed point.

## 1 Introduction

The study of frictional contact problems involving piezoelectric materials has been a topic of considerable interest in various industrial applications as well as in daily life in recent years. These materials are extensively employed in sensor and actuator technologies due to their exceptional ability to couple electrical and mechanical displacements, which means they can alter electrical polarization when subjected to mechanical stress or undergo mechanical strain when exposed to an electric field.

Over the recent years, thermo-piezoelectric and piezoelectric frictional contact problems with or without a conductive foundation have been investigated in a large number of papers. Indeed, for the piezoelectric models, we refer the reader to see the works [4–6], and the references therein, while for thermo-piezoelectric models, we refer to [1–3]. In the article referenced as [6], a problem of static frictional contact between a piezoelectric body and a conductive foundation is presented by the authors. They have utilized variational inequalities and fixed point theory to demonstrate the existence and uniqueness of weak solutions. In [1], a Signorini contact problem in thermo-piezoelectricity with Tresca's friction law has been studied in analogous way.

Our purpose in this paper is to study the process of frictional contact between a thermo-piezoelectric body and a rigid thermally conductive foundation. The frictional contact is modeled with temperature dependent Signorini's conditions and a version of non-local Coulomb's friction law with slip dependent coefficient of friction. The weak variational formulation which consists of a system coupling a variational inequality for displacement field, an elliptic variational equality for the potential and variational inequality for the temperature is presented. Then, an existence and uniqueness result to the model is provided.

## 2 Physical model and its mathematical formulation

The physical setting of the contact problem is as follows. We consider a piezoelectric body occupying, in its reference configuration, an open and bounded domain $\Omega \subset \mathbb{R}^d$, $d = 2,3$ with a sufficiently smooth boundary $\partial\Omega = \Gamma$. This boundary is divided into three open disjoint parts $\Gamma_1$, $\Gamma_2$ and $\Gamma_3$, on the one hand, and a partition of $\Gamma_1 \cup \Gamma_2$ into two open parts $\Gamma_a$ and $\Gamma_b$, on the other hand, such that $meas(\Gamma_1) > 0$ and $meas(\Gamma_a) > 0$. The body is supposed to be stress free at a free temperature and the temperature variations, accompanying the deformations, produce changes in the material parameters which are considered as depending on temperature. The body is clamped on $\Gamma_2$ and is subjected to a volume force $f_0$ in $\Omega$, a surface tractions of density $f_2$ act on $\Gamma_2$, a volume electric charge $\phi_0$ on $\Omega$, a surface electric charge of density $\phi_b$ is prescribed on $\Gamma_b$ and heat source $q_0$. The electric potential vanishes on $\Gamma_a$ and the temperature is assumed to zero on $\Gamma_a \cup \Gamma_b$. Moreover, on $\Gamma_3$ the body is in contact with friction with a thermally conductive obstacle, the so-called foundation. We model the contact with the Signorini contact conditions and friction.

Here and below, we do not indicate the dependence of various functions on the spatial variable $x \in \bar{\Omega}$, the indices $i, j, k, l$ take values between 1 and $d$, the summation convention over repeated indices is used and the index that follows a comma indicates a partial derivative with respect to the corresponding component of the spatial variable $u_{i,j} = \frac{\partial u_i}{\partial x_j}$. We denote by $\mathrm{Div}\,\sigma = (\sigma_{ij,j})$, $\mathrm{div}\,\mathrm{D} = (\mathrm{D}_{j,j})$ the divergence operator for tensor and vector valued functions, respectively. Also, we denote by $\mathbb{S}^d$ the space of second order symmetric tensors on $\mathbb{R}^d$ and $\nu$ represent the unit outward normal on $\Gamma$. Furthermore, we use the notation $u_\nu$ and $u_\tau$ for the normal and tangential displacement that is $u_\nu = u \cdot \nu$ and $u_\tau = u - u_\nu \nu$. We also denote by $\sigma_\nu$ and $\sigma_\tau$ the normal and tangential tress give by $\sigma_\nu = \sigma\nu \cdot \nu$, $\sigma_\tau = \sigma\nu - \sigma_\nu \nu$.

The elastic strain-displacement, electric field-potential and thermal field-temperature change relations are given by:

$$\varepsilon(u) = \frac{1}{2}\left(\nabla u + \nabla u^*\right), \quad \mathrm{E}(\varphi) = -\nabla\varphi, \quad q = -\mathcal{K}\nabla\theta \quad \text{in } \Omega,$$

where $u : \Omega \to \mathbb{R}^d$, $\varepsilon(u) = (\varepsilon_{ij}(u))$, $\sigma : \Omega \to \mathbb{S}^d$, $\varphi : \Omega \to \mathbb{R}$, $\mathrm{E}(\varphi) = (\mathrm{E}_i(\varphi))$, $\theta : \Omega \to \mathbb{R}$, $\mathrm{q} : \Omega \to \mathbb{R}^d$ and $\mathcal{K} : \Omega \times \mathbb{R}^d \to \mathbb{R}^d$, are, respectively, the displacement field, the linear strain tensor, the stress tensor, the electrical potential, the electric vector field, the temperature, the heat flux vector and the thermal conductivity tensor. The equilibrium equations and the constitutive relations form the governing equations, and in the case of a static process, they can be expressed as follows:

$$\mathrm{Div}\,\sigma + f_0 = 0, \quad \mathrm{div}\,\mathrm{D} = q_0, \quad \mathrm{div}\,q = \phi_0, \quad \text{in } \Omega, \tag{1}$$

where $\mathrm{D} : \Omega \to \mathbb{R}^d$ is the electric displacements field. The constitutive equations of a nonlinear piezoelectric material including the effect thermal expansion can be written as:

$$\sigma = \mathfrak{F}\varepsilon(u) - \mathcal{E}^*\mathrm{E}(\varphi) - \mathcal{M}\theta, \qquad \mathrm{D} = \mathcal{E}\varepsilon(u) + \beta\mathrm{E}(\varphi) + \mathcal{P}\theta, \quad \text{in } \Omega, \tag{2}$$

in which $\mathfrak{F} : \Omega \times \mathbb{S}^d \to \mathbb{S}^d$, $\mathcal{E} : \Omega \times \mathbb{S}^d \to \mathbb{R}^d$, $\beta : \Omega \times \mathbb{R}^d \to \mathbb{R}^d$, $\mathcal{M} : \Omega \times \mathbb{R} \to \mathbb{S}^d$, $\mathcal{P} : \Omega \times \mathbb{R} \to \mathbb{R}^d$, are respectively, the non linear elasticity operator, the piezoelectric tensor, the linear electric permittivity operator, the thermal stress operator, the pyroelectric operator. $\mathcal{E}^*$ is the transpose of $\mathcal{E}$ given by $\mathcal{E}^* = (e_{kij})$ and satisfies $\mathcal{E}\sigma \cdot v = \sigma \cdot \mathcal{E}^* v$ for all $\sigma \in \mathbb{S}^d$, $v \in \mathbb{R}^d$. Here, we suppose that the nonlinear elasticity operator is the one that describes the behavior of Hencky's materials. Hence, the stress-strain relation is given by:

$$\mathfrak{F}\varepsilon(u) = k_0 \operatorname{tr}(\varepsilon(u))\mathrm{I} + 2g(\|\bar{\varepsilon}(u)\|^2)\bar{\varepsilon}(u) \quad \text{in } \Omega.$$

with $k_0 > 0$ a material coefficient, I the identity tensor of second order, $\operatorname{tr}(\varepsilon) = \varepsilon_{ii}$ the trace of $\varepsilon$ and $\bar{\varepsilon}$ denotes its deviatoric part: $\bar{\varepsilon} = \varepsilon - \frac{1}{d}\operatorname{tr}(\varepsilon)\mathrm{I}$.

Next, to complete the mathematical model, according to the description of the physical setting, we consider the followings boundary conditions:

$$u = 0 \text{ on } \Gamma_1, \quad \sigma\nu = f_2 \text{ on } \Gamma_2, \quad \varphi = 0 \text{ on } \Gamma_a, \quad \mathrm{D}\cdot\nu = \phi_b \text{ on } \Gamma_b, \quad \theta = 0 \text{ on } \Gamma_1 \cup \Gamma_2. \qquad (3)$$

On the contact surface $\Gamma_3$, we consider:

$$\sigma_\nu(u, \varphi, \theta) \leq 0, \quad u_\nu \leq 0, \quad \sigma_\nu(u, \varphi, \theta)u_\nu = 0 \qquad \text{on } \Gamma_3, \qquad (4)$$

$$q_\nu(u, \varphi, \theta) \leq 0, \quad (\theta - \theta_F) \leq 0, \quad q_\nu(u, \varphi, \theta)(\theta - \theta_F) = 0 \qquad \text{in } \Gamma_3, \qquad (5)$$

$$\|\sigma_\tau\| \leq \mu(\|u_\tau\|)|\mathrm{R}\sigma_\nu(u, \varphi, \theta)|, \begin{cases} \|\sigma_\tau\| < \mu(\|u_\tau\|)|\mathrm{R}\sigma_\nu(u, \varphi, \theta)| \Rightarrow u_\tau = 0, \\ \sigma_\tau = -\mu(\|u_\tau\|)|\mathrm{R}\sigma_\nu(u, \varphi, \theta)| \dfrac{u_\tau}{\|u_\tau\|} \Rightarrow u_\tau \neq 0, \end{cases} \qquad \text{on } \Gamma_3, \qquad (6)$$

Conditions (4)-(5) represent Signorini contact conditions for the displacement and temperature fields, while condition (6) represents Coulomb's friction law in which $\mu$ is the coefficient of friction and R is a regularization operator.

We use the above equations and conditions to obtain the following mathematical problem.

**Problem (P).** Find a displacement field $u : \Omega \to \mathbb{R}^d$, a stress field $\sigma : \Omega \to \mathbb{S}^d$, an electric potential $\varphi : \Omega \to \mathbb{R}$, an electric displacement field $\mathrm{D} : \Omega \to \mathbb{R}^d$, a temperature field $\theta : \Omega \to \mathbb{R}$ and a heat flux $q : \Omega \to \mathbb{R}^d$, satisfying (1)-(6).

To establish the unique solvability of our problem, we first introduce functional spaces for various quantities. We then state assumptions about the given data and present the variational formulation associated with the problem. Furthermore, we state and prove our main result, the existence of a unique weak solution to the model. The proofs are based on arguments from elliptic variational inequalities and Banach fixed point properties of certain maps.

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# On the $S$-packing coloring of circulant graphs $C_n(1,t)$

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**Abstract**

Let $S=(a_1,a_2,\ldots)$ be a non-decreasing sequence of positive integers. Given a graph $G$, an $S$-packing $k$-coloring of $G$ is the mapping $f:V(G)\to\{1,\ldots,k\}$ such that every two distinct vertices $u$ and $v$ with $f(u)=f(v)=i$ are at pairwise distance at least $1+a_i$. The $S$-packing chromatic number, denoted by $\chi_S(G)$, is the smallest $k$ such that $G$ admits an $S$-packing $k$-coloring. If $S=(1,2,3,\ldots)$, the number $\chi_S(G)$ is known as the packing chromatic number and denoted by $\chi_\rho(G)$.

Let $D=\{d_1,\ldots,d_k\}$ be a finite set of positive integers. The distance graph graph $G(\mathbb{Z},D)$ is the infinite graph with vertex set $\mathbb{Z}$ and two distinct vertices $i$ and $j$ are adjacent if and only if $|i-j|\in D$. The circulant graph $C_n(d_1,\ldots,d_k)$ is the finite graph with vertex set $\mathbb{Z}/n\mathbb{Z}$ and two distinct vertices $i$ and $j$ are adjacent if and only if $|j-i|\equiv d_l \bmod n$ for some $d_l$ in $D$.

Some results for $S$-packing colorings of $C_n(2,t)$ are given in [1]. In this paper we establish the the $S$-packing coloring of $C_n(1,t)$ if $t\in\{2,3,4\}$ and partial results for arbitrary odd $t$.

*Keywords :* Circulant graphs, $S$-packing coloring, $S$-packing chromatic number

## 1 Introduction

Let $D=\{d_1,\ldots,d_k\}$ be a finite set of positive integers. The distance graph graph $G(\mathbb{Z},D)$ is the infinite graph with vertex set $\mathbb{Z}$ and two distinct vertices $i$ and $j$ are adjacent if and only if $|i-j|\in D$. Similarly, the circulant graph $C_n(d_1,\ldots,d_k)$ is the finite graph with vertex set $\mathbb{Z}/n\mathbb{Z}$ and two distinct vertices $i$ and $j$ are adjacent if and only if $|j-i|\equiv d_l \bmod n$ for some $d_l$ in $D$.

Togni [5] was the first to investigate the packing coloring of distance graphs where he focused on the case $1\in D$ and other papers continued the study of these graphs [2–4]. Recently, Bresar *et al.* [1] considered the $S$-packing coloring of the distance graph $G(\mathbb{Z},\{2,t\})$ and gave some results for $S$-packing colorings of $C_n(2,t)$. In this paper we establish the the $S$-packing coloring of $C_n(1,t)$ if $t\in\{2,3,4\}$ and partial results for arbitrary odd $t$.

## 2 $C_n(1,2)$

For the sequence $S=(1,2,\ldots,k)$, a periodic packing coloring of period 54 of the distance graph $D(1,2)$ using colors from $\{1,\ldots,8\}$ was given in [5]. Hence $\forall n\equiv 0 \bmod 54$, $\chi_\rho(C_n(1,2))\le 8$. Actually, computations on the first values of $n$ lead us to think that if $n$ is not a multiple of 54, then 8 colors are no more sufficient for a packing coloring and that 9 colors are sufficient in general.

**Proposition 2.1.** *For all $n \geq 54$ , $\chi_\rho(C_n(1,2)) \leq 9$.*

**Proposition 2.2.** *For any integer $n \geq 20$, the circulant graph $C_n(1,2)$ is $(1,2,2,2,2)$-packing colorable.*

Moreover, we have checked by computer that $C_{19}(1,2)$ is not $(1,2,2,2,2)$-packing colorable, hence the lower bound on $n$ in the above proposition is minimal.

**Proposition 2.3.** *For any integer $n \geq 27$, the circulant graph $C_n(1,2)$ is $(1,1,3,3,3)$-packing colorable.*

Again, we have checked by computer that $C_{26}(1,2)$ is not $(1,1,3,3,3)$-packing colorable and that $C_{13}(1,2)$ is not $(1,1,3,3,3,3)$-packing colorable.

## 3 $C_n(1,3)$

For the sequence $S = (1,2,\ldots,k)$, it was shown in [5] that $\chi_\rho(D(1,3)) = 9$ and a periodic coloring of period 32 of the distance graph $D(1,3)$ using colors from $\{1,\ldots,9\}$ was given. Hence, for all $n \equiv 0 \bmod 32$, $\chi_\rho(C_n(1,3)) \leq 9$.

**Conjecture 1.** *For $n \geq 36$, we have $\chi_\rho(C_n(1,3)) \leq 9$.*

**Proposition 3.1.** *For $n \geq 32$, we have $\chi_\rho(C_n(1,3)) \leq 12$.*

Remark that if $n$ is even, then $C_n(1,3)$ is bipartite, i.e., $(1,1)$-packing colorable.

**Proposition 3.2.** *For any integer $n \geq 28$, the circulant graph $C_n(1,3)$ is $(1,2,2,2,2)$-packing colorable.*

We have checked by computer that $C_{27}(1,3)$ is not $(1,2,2,2,2)$-packing colorable.

**Proposition 3.3.** *For any odd integer $n \geq 7$ and $k \geq 2$, the circulant graph $C_n(1,3)$ is $(1,1,k,k,k)$-packing colorable.*

## 4 $C_n(1,4)$

For the sequence $S = (1,2,\ldots,k)$, a periodic coloring of period 90 of the distance graph $D(1,4)$ using colors from $\{1,\ldots,14\}$ was given in [5]. Hence $\forall n \equiv 0 \bmod 90$, $\chi_\rho(C_n(1,4)) \leq 14$.

**Conjecture 2.** *For any integer $n \geq 24$, the circulant graph $C_n(1,4)$ is $(1,1,2,2)$-packing colorable.*

**Conjecture 3.** *For any integer $n \geq 30$, the circulant graph $C_n(1,4)$ is $(1,1,3,3,3)$-packing colorable; and not $(1,1,3,3)$-packing colorable if $n \not\equiv 0 \bmod 10$.*

**Proposition 4.1.** *For any integer $n \geq 30$ :*

1. *The circulant graph $C_n(1,4)$ is $(1,1,3,3)$-packing colorable if $n \equiv 0 \bmod 10$.*
2. *The circulant graph $C_n(1,4)$ is $(1,1,3,3,3)$-packing colorable if $n \not\equiv 0 \bmod 10$.*

## 5 $C_n(1,t)$ ($t$ is odd)

If $n$ is even and $t$ is odd, we prove that the circulant graph $G = C_n(1,t)$ is bipartite.

**Proposition 5.1.** *For all even integer $n \geq 6$ and all odd integer $3 \leq t \leq \frac{n}{2}$, the circulant graph $C_n(1,t)$ is $(1,1)$-packing colorable.*

**Proposition 5.2.** *For all odd integer $n \geq 7$ and all odd integer $3 \leq t \leq \left\lfloor \frac{n}{2} \right\rfloor$, the circulant graph $C_n(1,t)$ is $(1,1,2,2,2)$-packing colorable.*

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# A Genetic Algorithm Resolution for the CETSP problem

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**Abstract**

We address a variant of the Traveling Salesman Problem known as the Close-Enough Traveling Salesman Problem (CETSP). In this problem, if a salesman is within a specified distance of a node, then the node has been visited.To solve the CETSP, we propose in this work to use a genetic algorithm that we have designated as (GACETSP). In detail, The new proposed algorithm will be declined in two versions $[GACETSP]_{randtour}$ and $[GACETSP]_{inttour}$ to be able to test and evaluate two resolution strategies proposed for the solution of our problem. Then, we analyze the results provided by these two versions by comparing them with results existing in the literature . Overall, our method is very fast and improves upon heuristics from the literature.

*Keywords :* CETSP; TSP; Genetic algorithm; Optimization

## 1 Introduction

The Close-Enough Traveling Salesman Problem (CETSP) [2,4] is a generalization of the TSP, in which the salesman does not need to visit the exact location of each customer. Instead, a compact region of the plane containing each node is specified as its neighborhood set, and the goal is to find a shortest tour that starts from a specified depot location and intersects all of these neighborhood sets.
Intuitively speaking, if a salesman is within a specified distance of a node, then the node is considered to have been visited. In the CETSP, the salesman moves freely in 2D Euclidean space, whereas, in the TSP, the salesman travels from node to node.
More precisely, we search for the tour T in a continuous space instead of looking for the tour in a graph G.

The CETSP has many applications in real-world problems For example, by using Radio Frequency Identification (RFID) tags connected to physical meters one can encode the identification number of the meter and its current reading into digital signals.This way, an utility truck equipped with an Automatic Meter Reading (AMR) system can remotely collect and transmit data from a certain distance.
The paper is structured as follows. The problem and the notation used are described in Section 2. Section 3 is devoted to the solution approach.

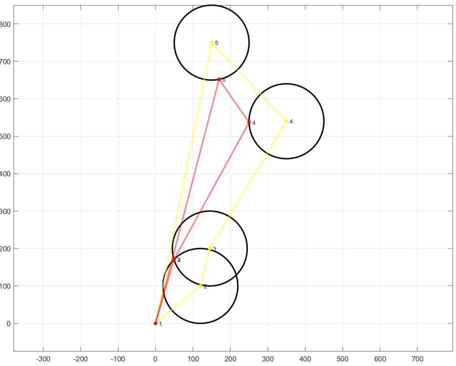

Figure 1: The optimal TSP and CETSP tour.

## 2 Problem Formulation

Let $C = \{C_0, \ldots, C_n\}$ be a CETSP-partitioning of the plane with a pairwise distance matrix $L = \{l_{ij}\}$. For each $m \in M$ define the set of cells intersecting $S_m$ as $N(m) = \{1 \leq i \leq n : C_i \cap S_m \neq \emptyset\}$. we have $N(m) \neq \emptyset$ for all $m \in M$. Consider the following MIP:

$$\min \sum_{i=0}^{n} \sum_{j=0}^{n} l_{ij} x_{ij} \tag{1}$$

$$\text{s.t.} \sum_{j=0}^{n} x_{ij} = \sum_{j=0}^{n} x_{ji}, \quad \forall i = 0, \ldots, n, \tag{2}$$

$$y_i = \sum_{j=0}^{n} x_{ji}, \quad \forall i = 0, \ldots, n \tag{3}$$

$$\sum_{i \in N(m)} y_i \geq 1, \quad \forall m \in M, \tag{4}$$

$$\sum_{i \in S} \sum_{j \notin S} x_{ij} \geq y_v, \quad \forall S \subset \{1, \ldots, n\}, \quad 2 \leq |S| \leq |C| - 2 \text{ and } v \in S, \tag{5}$$

$$x_{ij} \in \{0, 1\}, \quad \forall i = 0, \ldots, n, j = 0, \ldots, n \tag{6}$$

$$0 \leq y_i \leq 1, \quad \forall i = 1, \ldots, n; \quad y_0 = 1. \tag{7}$$

The objective function (1) minimizes the total distance traveled in the tour. Constraint (2) ensure that for each cell $C_i$, the number of incoming tour arcs equals the number of outgoing tour arcs. constraint (3) define $y$ variables in terms of $x$ variables. (In fact, the formulation can be given without the y variables; they are included only for convenience in presentation.) Constraint (4) ensure that for each $m \in M$, at least one element of C is visited that covers $m$. Constraint (5) are subtour elimination constraints.

## 3 Resolution

Genetic algorithm (GA) is a stochastic search algorithm. GA is inspired from biological evolution process. It was first used by John Holland in 1992 [3] and have been widely used to solve combinatorial optimization problem. A genetic algorithm is a successor to the traditional evolutionary algorithm where at each step it will select random solutions from the present population and labels those as parents and uses them to reproduce to the next generation as children with a series of biological operations, namely reproduction, selection, crossover and mutation. More recently, GA was proposed to tackle CETSP [1]. In our work, we use the real coding of the solutions since this type of coding is the most adapted to the problems, and we give a specified choice of the crossover and mutation operators.

| R-Instance | Size | $T_0$ | $AGCETSP_{randtour}$ r= 100 | | $AGCETSP_{randtour}$ r= 200 | | $AGCETSP_{randtour}$ r=250 | |
|---|---|---|---|---|---|---|---|---|
| | | | $T_1$ | Time(s) | $T_1$ | Time(s) | $T_1$ | Time(s) |
| CETSP-5 | 5 | **2965.3** | 2595 | **1.5303** | 2323 | **1.5056** | **2247** | **1.4902** |
| CETSP-10 | 25 | 4502 | 3279 | 3.6324 | 3224 | 3.6134 | **2930** | 3.6816 |
| CETSP-75 | 75 | 16860 | 11741 | 10.4615 | 11322 | 10.5151 | **11008** | 10.052 |
| CETSP-100 | 100 | 28232.39 | 19542 | 20.1176 | 16879 | 20.0059 | **10159** | 20.5423 |
| CETSP-200 | 200 | 65570.10 | 48435 | 44.8667 | 44137 | 44.7469 | 42517 | 44.5740 |
| CETSP-300 | 500 | 111908.9 | **87450** | **59.8465** | **79686** | **57.9902** | **77298** | **58.2934** |

## 4 Conclusion and perspectivs

This paper has presented a new, straightforward genetic algorithm for solving the CETSP.
The proposed algorithm assigns a good tour sequence to them, and then minimizes the tour's length . We compared a different heuristics, including seven from the literature, for solving the CETSP on many test problems. We found that the combination of low computation time and high solution quality made the genetic algorithm very competitive methods. Our future work will apply our heuristics to problems with an arbitrary radius for each node. We will also solve problems with 3D Euclidean reduce the number of points needed, in order to determine a series of dimension-dependent values for the number of points considered, in order not to complicate the parameterizations of the proposed metaheuristic.

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# Alzheimer disease based Artificial Intelligence diagnosis: short review and future trends

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**Abstract**

Alzheimer's disease is a degenerative calcifying brain disease defined as a progressive impairment of brain cells, is a major form of dementia that has recently received much attention in neuro-imaging techniques and poses a serious problem in modern health care. The disease affects more than 45 million people worldwide and according to research, will double in the next 20 years. This can affect cognitive function (thinking ability) and mental function (emotional and behavior) over time and can lead to a continual decline in memory. With no cure for the disease with current therapies, early diagnosis is the only option to reduce the severity of AD and enable patients to live a good quality of life.

The main goal is to use different types of biomarkers to identify dementia in a variety of people, diverse analytical and evaluation techniques performed in recent studies on early detection of AD, exploring the role of emerging technologies, such as machine learning approach and deep learning models and computer vision technique that utilize Biomarker Methods, Fusion, and Registration for multimodality, to pre-process medical scans.

We provide an overview of the current state of AD diagnosis and highlights the potential future trends, that could revolutionize the field, and the advantages of AI in AD diagnosis, it has shown promising result in differentiating AD from other forms of dementia, predicting disease progression and assisting in personalized treatment, and the integration of AI algorithms into clinical decision support systems for real-time diagnosis and monitoring.

*Keywords :* Alzheimer's disease (AD), Dementia, Machine Learning, Artificial intelligence, Medical imaging

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